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SUPERSONIC DIFFUSERS

by

J. LUKASIEWICZ, B.Sc., D.I.C.

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ROYAL AIRCRAFT ESTABLISHMENT, FARNBOROUGH

Supersonic Diffusers

- by -

J. Lukasiewicz, B.Sc., D.I.C.

R.A.E. Ref: Gas/3103/JL/127.

SUMMARY

Various possible types of supersonic diffuser are considered theoretically and the available experimental evidence is reviewed.

The most obvious type of an efficient supersonic diffuser is the reversed supersonic nozzle. A stability criterion for the reversed supersonic nozzle is developed, which shows that the contraction between the entry and the throat is so limited, that if a normal shock at entry is to be avoided, such a diffuser is of little practical use unless an artificial means of inducing and maintaining the less stable flow can be found.

Test results of simple pitot entries show that, in the absence of the boundary layer, the one-dimensional theory holds for normal shocks. Since their efficiency is high for Mach No. up to about 1.5, diffusers with normal shock at entry are quite satisfactory in this velocity range.

The mechanism of the shock wave and boundary layer interaction is considered. When normal shocks occur in the presence of the boundary layer, as in annular pitot entries and diffusers for supersonic wind tunnels, their compression efficiency is appreciably lower than the theoretical one.

For very high velocities, exceeding M = 2.5, the only type of diffuser which is practicable and offers high theoretical efficiency is the multi shock diffuser. The efficiencies of this type of diffuser are examined and shown to exceed 90% up to M = 3.0 for designs having three or four oblique shocks. Various geometrical arrangements are considered from the point of view of reduction of frontal area for a given mass flow and considerable improvements on the single focussed wave system are obtained.

The high performance of multi shock diffusers is confirmed by Oswatitsch's tests at Göttingen, the results of which are here summarised.

The operation of supersonic diffusers at other than the design Mach No. is briefly considered.

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1 Introduction

The problem of diffusion from supersonic velocity is of primary importance in the design of power plants for supersonic aircraft, which utilize the atmospheric air as the working substance. The fundamental requirements to be satisfied by diffusers in general are high efficiency of the kinetic energy conversion and small drag coefficient. In order to meet those requirements, entirely different methods from those suitable at subsonic speeds must be used at supersonic velocities.

One of the basic characteristics of supersonic diffusers as distinct from the subsonic ones is that the maximum possible flow through them is determined, at any supersonic speed, by the diffuser frontal area and is equal to the corresponding swept volume.

Further, unlike subsonic flow, a continuous diffusion from supersonic velocity by means of a convergent-divergent duct is, in practice, impossible. In other types of theoretically isentropic supersonic diffusers the compression actually occurs through a number of finite shock waves, so that a system of shock waves must always be present and the design problem is largely that of determining the optimum system to be used. It seems that a strict mathematical analysis, which would define the thermodynamically most efficient shock wave formation for any given set of conditions, would not lead to results of practical significance. Instead, the performance of shock systems, the design of which is based on simple criteria, can be easily estimated. Such analysis also shows that the differences in the efficiencies achieved when various criteria are used, are small and that it is mainly the number of shocks which determines the efficiency, for given initial and final conditions.

The geometrical principles of an efficient shock system are deduced from the examination of a number of configurations and are in general the same for two-dimensional and axially-symmetrical diffusers; the actual diffuser shapes are however different, as determined by the appropriate solutions for supersonic flow.

The subject of supersonic diffusers is reviewed in this paper on the above lines. The calculations were made for flow of air and, in the case of multi-shock diffusers, for two-dimensional flow. Results of the same order would, however, also be obtained for axially-symmetrical flow, so that the general conclusions apply also to this type of diffuser.

In supersonic flow theory the gas is usually assumed to be perfect, with constant specific heats, and the viscosity and thermal conductivity are neglected. The actual performance of diffusers depends however to a large extent on the effects of friction and the boundary layer, which can only be ascertained experimentally. The available experimental results on those effects in supersonic flow, are summarized in this report but are not adequate, especially in the case of multi-shock diffusers and at very high velocities, to form a basis for design and further research is needed.

It is usual to evaluate the diffuser performance in terms of "compression efficiency". In the technical literature several ratios are used to denote the efficiency of compression and they are collected and defined in the Appendix, together with most of the symbols used in this report. For our purpose, since the compression in the diffusers considered is adiabatic, the isentropic efficiency η_σ, (A1)*, is the appropriate one to use. As shown in the Appendix, the isentropic

* Numbers preceded by A refer to the equations of the Appendix.

efficiency η_σ is a function of the Mach Number M_1 and the ratio p_σ of the isentropic stagnation pressures before and after the diffusion, or the entropy increase Δ_σ.

2 Reversed flow in a supersonic nozzle

Before considering more complex forms of supersonic diffusers, the simple case of flow in a convergent-divergent nozzle, with supersonic flow velocity at entry, will be discussed. Such a discussion yields important results, both from the theoretical and practical points of view.

2.1 General discussion

Consider the flow through a system like that shown in fig.1, consisting of two nozzles joined by a constant area duct. The entry nozzle serves to obtain a certain constant Mach Number $M > 1.0$, whereas the second (reversed) nozzle throat area $A_{t''}$ can be varied. The gas starts to flow through the system from zero velocity and pressure P_0 at section 'c' and leaves the system at section 'c' at a pressure P_0, which will be also referred to as the back pressure.

The discussion of flow in such a system applies to certain types of supersonic wind tunnels and, by discarding the first nozzle, it is extended to diffusers for supersonic power plants.

The flow will be assumed to be one-dimensional and, except for normal shocks, isentropic. For isentropic flow the relationship between the Mach Number M and the cross-sectional area ratio (contraction) ψ corresponding to a change of velocity to the sonic value ($M = 1.0$) is given by

$$\psi = M\left\{(\gamma + 1)\Big/\left[2 + (\gamma - 1)M^2\right]\right\}^{\frac{\gamma+1}{2(\gamma-1)}} \qquad (1)$$

as obtained from the de Saint Vénant and Wantzel equations for the velocity and rate of flow at constant entropy. Referring to fig.1, if $\psi = A_{t'}/A$ and $M_{t'} = 1.0$, then M is given by function (1), which is drawn for air in fig.3, curve 1. It will be noticed that to each value of ψ there correspond two velocities, one sub- and the other - supersonic.

It will be remembered that any losses (i.e. entropy increase) have the effect of increasing the cross-sectional area at which a given velocity (or temperature) is obtained as compared with the isentropic flow, the corresponding pressure and density being smaller. Also, the losses due to a normal shock increase with the velocity on the upstream side of the shock (cf. fig.9, 10).

When the second throat is smaller than the first one ($A_{t''} < A_{t'}$), no supersonic flow is possible upstream of t". If a diffuser of such a contraction were inserted in a supersonic stream of velocity M, as determined by the first nozzle, a detached shock front would form at its entry, the flow being smaller than that corresponding to the swept volume at velocity M.

With equal throat areas ($A_{t''} = A_{t'}$), the flow between the throats can be theoretically either sub- or supersonic. Any losses, however, would make it impossible for supersonic flow to develop upstream of t", as in the case previously considered. Further, it seems that the possibility of attainment of supersonic velocity between the throats depends on the manner in which the flow is initiated. If the system of fig.1 were instantaneously connected to a vacuum, supersonic velocity would be developed upstream of t". But, on the other hand, if the back pressure P_o were gradually reduced from the value P_o', the flow would start as subsonic throughout and would presumably remain subsonic upstream of t".

It follows from the above that in practice in order to obtain a stable supersonic flow upstream of the second throat, whether the jet is constrained or free, the second throat must be larger than the first one, so that a shockless diffusion from supersonic velocity by means of a convergent-divergent duct becomes impossible. A closer investigation will indicate the necessary increase in the throat cross-section.

Let $A_{t''} > A_{t'}$, and pressure P_o be gradually reduced. For sufficiently low P_o value sonic flow velocity is obtained in the first throat, the flow being subsonic throughout. With lower back pressures supersonic flow develops past t' and a normal shock forms in the divergent part of the first nozzle and moves downstream as the back pressure is reduced. For given initial and final conditions the shock could form either in the first or in the second nozzle, at equal cross-sections. However, it is known from the supersonic tunnel operation that the shock actually occurs always in the first nozzle. When the shock reaches the maximum cross-section A and provided the flow remains subsonic at the second throat, a further decrease in the back pressure will cause the shock to jump downstream of t", to a cross-section larger than A, the flow becoming eventually supersonic throughout.

If the second throat were made rather smaller, but still larger than the first, so that with the shock present at some cross-section between the two throats, sonic velocity were again reached at t", the sequence of events would be different from that described above. A further decrease in the back pressure would have now no influence on the flow upstream of t", but supersonic flow and a second shock would form downstream of t"; the second shock would move towards the exit as the pressure were decreased*.

The possibility of such a flow configuration leads to a number of important conclusions.

2.2 Limiting contraction

It has been already remarked that, in order to achieve a diffusion more efficient than that corresponding to a normal shock at the free-stream Mach Number, the velocity must be supersonic at the diffusor throat and a shock must occur downstream. The nearer to the second throat the shock takes place, the more efficient is the compression. The possibility of a flow configuration with two shocks limits the minimum size of the second throat and thus effects the maximum compression efficiency of a "reversed nozzle" diffuser.

Let the system of fig.1 represent the nozzle, working section and diffuser of a supersonic tunnel which is started by a gradual creation of pressure difference (say a closed-circuit tunnel). The second throat must be sufficiently large to prevent the flow from reaching sonic velocity and hence to allow the shock to jump past it, so that stable supersonic flow can be established in the working section. The minimum second throat size must be therefore such that, with shock occuring in the working section (at M), the velocity in the second throat is just sonic. The maximum permissible back pressure required to start a tunnel of this type is smaller than that

* These flow configurations are described in ref.2 and were observed by Seippel, who presented photographs of analogous flows of water in a shallow channel (ref.15).

corresponding to a normal shock in the working section. Once the tunnel is started, the back pressure can be increased, the shock advancing towards the throat but always remaining on its downstream side.

Theoretically such difficulties would not be encountered if the tunnel were connected, prior to starting, to a highly evacuated container and the pressure difference applied instantaneously.

Disregarding the first nozzle, a similar argument applies to a supersonic diffusor of the reversed nozzle type inserted in a free stream of velocity M.

When the contraction of the diffuser ψ is equal to or smaller than that which gives sonic flow velocity at the throat when a normal shock forms at entry, then theoretically two flow configurations are possible, as described before. Experiment shows that the less efficient flow pattern, with normal shock at entry, is by far the more stable one.

From these considerations it is evident that in order to achieve a diffusion by means of a reversed nozzle with an efficiency higher than that of a normal shock at the free-stream velocity, the possibility of a flow configuration which includes such a shock must be eliminated by making the throat sufficiently large. The same condition must also be satisfied to make the starting of certain types of supersonic wind tunnels possible.

The limiting value of the contraction ratio $\psi_{min} = A_{t^*}/A$ is obtained by assuming a normal shock at the entry and sonic velocity at the throat. We have for the normal shock

$$M' = \left[\left(M^2 + \frac{2}{\gamma - 1}\right)/\left(\frac{2\gamma}{\gamma - 1} M^2 - 1\right)\right]^{\frac{1}{2}} \tag{2}$$

where M' is the Mach Number on the downstream side of the shock. Combining this with relation (1), the following is obtained for the limiting contraction ψ_{min}:

$$\psi_{min} = \left(\frac{\gamma - 1}{\gamma + 1}\right)^{\frac{\gamma+1}{2(\gamma-1)}} \sqrt{\frac{2}{\gamma - 1}} \left(\frac{2\gamma}{\gamma - 1} - \frac{1}{M^2}\right)^{\frac{1}{\gamma-1}} \left(\frac{\gamma - 1}{2} + \frac{1}{M^2}\right)^{\frac{1}{2}} \tag{3}$$

This function has been calculated for $\gamma = 1.400$. Its values are given in Table I and it is drawn in fig.3, curve 2*.

---

* It appears that the same criterion for the limiting contraction has been derived by A. Kantrowitz and Coleman du P. Donaldson in a N.A.C.A. paper entitled "Preliminary investigation of supersonic diffusors", February 1945, to which reference is made by Troller, ref.18. However, the results, as quoted by Troller, do not agree with our equation (3). The limiting contraction ratio is given as

$$\psi_{min} = \frac{A_{min}}{A_{intake}} = \frac{\left(\frac{\gamma + 1}{\gamma - 1}\right)^{\frac{\gamma+1}{\gamma-1}} \cdot M^{\frac{3\gamma-1}{\gamma-1}}}{\left(\frac{\gamma - 1}{2}\right)^{\frac{1}{2}\frac{\gamma+1}{\gamma-1}} \left(\frac{2}{\gamma - 1} + M^2\right)^{\frac{3\gamma+1}{\gamma-1}} \left(\frac{2\gamma}{\gamma - 1} M^2 - 1\right)^{\frac{1}{\gamma-1}}}$$

but it does not agree with the tabulated values of M and ψ_{min} (presumably for $\gamma = 1.403$), which however compare fairly well with our table I.

In ref.18 also the values of the compression efficiency (presumably isentropic efficiency, our η_σ) are tabulated; they are slightly higher than ours, given in table II, p.14.

Table I ($\gamma = 1.400$)

M	ψ_{min}
1.0	1.000
1.2	0.978
1.6	0.894
2.0	0.823
2.5	0.760

M	ψ_{min}
3.0	0.720
3.5	0.692
4.0	0.672
4.5	0.657
5.0	0.648
∞	0.600

From expression (3) the asymptotic character of the ψ_{min} function is evident. At an infinite Mach Number

$$\left(\psi_{min}\right)_{M=\infty} = \left(\frac{\gamma - 1}{\gamma + 1}\right)^{\frac{\gamma+1}{2(\gamma-1)}} \left(\frac{2\gamma}{\gamma - 1}\right)^{\frac{1}{\gamma-1}} \tag{4}$$

$$= 0.6002 \qquad \text{for } \gamma = 1.400.$$

2.3 Normal shock in absence of boundary layer: pitot type diffuser

(i) It is theoretically possible to design a diffuser of the reversed nozzle type, two-dimensional or axially symmetrical, in which the compression is isentropic, i.e. no shocks are formed. In both cases the correct wall profile is determined by the method of characteristics. Such a shockless two-dimensional diffuser is shown in fig. 2 (characteristics at 4^{o} intervals).

Nevertheless, the previous considerations indicate that a stable supersonic flow in such an ideal diffuser cannot be obtained and that a contraction $\psi > \psi_{min}$ must be used to avoid the formation of a shock front at the diffuser entry. Although equation (3), which expresses this condition, has been obtained on the simplest theoretical assumptions, it gives a very accurate quantitative representation of the actual phenomenon, as proved experimentally. Theoretically this would be expected from the following consideration of flow in a diffuser having a limiting contraction ratio ψ_{min}. The free-stream supersonic flow in front of the diffuser is truly one-dimensional. When a flow configuration with a shock at the diffuser entry develops, the shock, occuring in the absence of the boundary layer, conforms to the simple theory. Subsequent subsonic and accelerated flow in the converging part of the diffuser is known, as for subsonic nozzles, to be practically isentropic and the flow downstream of the throat does not effect equation (3).

On the other hand, when $\psi > \psi_{min}$ and the flow is supersonic at least down to the diffuser throat, the simple one-dimensional, isentropic flow theory cannot be expected to give accurate results. In practice weak shocks cannot be eliminated in the convergent part of the diffuser and normal shock, occuring inside the diffuser, is affected to a large extent by the boundary layer, as will be seen later.

(ii) It is well known that in front of a pitot tube the compression from supersonic velocity follows the simple normal shock theory. Further evidence on the compression through a normal shock has been obtained from tests (ref. 7, 8, 16, 19) of simple pitot type supersonic diffusers (fig. 4) and an excellent agreement with the simple theory has been found, both with respect to the efficiency and mass flow. As an example some results of ref. 7 are here tabulated:-

Free stream Mach Number		2.6
Mean Mach Number downstream of shock	theoretical	0.504
	observed	0.520
Mean static pressure ratio across shock	theoretical	7.70
	observed	7.77

The overall compression efficiency η_σ was found to be slightly lower than that corresponding to an isentropic subsonic compression. Values of $\eta_\sigma = 0.9$ (ref.8) and 0.84 (ref.19) were determined for the subsonic diffusion when a divergent conical diffuser of 7^o vertex angle was used. In ref.19 this efficiency was found to be constant over a range of entry Mach Numbers between 0.5 and 1.0. When the diffuser consisted of a parallel section at entry followed by a 10^o vertex angle conical duct, the subsonic compression efficiency was equal to approximately 0.75 (ref.8).

(iii) The effect of the variable diffuser contraction has been investigated at N.P.L., ref.7. A cylindrical glass tube was inserted in the supersonic air stream and blocked at the rear end by an adjustable cone. The degree of the blockage could be varied and the flow was observed at a constant free-stream Mach Number of 2.6.

According to the previous considerations, two different kinds of flow are possible with the above set-up, for sufficiently small exit areas. While decreasing the exit area supersonic flow was observed inside the tube until, when the blockage reached Ψ_{min} value, the flow, instead of remaining supersonic, suddenly changed to subsonic and a normal shock appeared at entry. When blockage was further increased, the flow remained subsonic but the shock left the tube entry, the mass flow through the tube being reduced and the velocity remaining sonic in the exit gap.

Similar experiments were carried out in Germany, at Göttingen (by Guderley) and at Völkenrode (cf. ref.9) and the results agree with the N.P.L. ones: the flow configuration with a normal shock at entry has been found to be by far the more stable one.

Attempts were made at Völkenrode to induce artificially the supersonic regime by "stirring up" the flow in the entry with a thin wire and thus disturbing the normal shock. It was found that in this way the efficient regime could be established and would even remain after the wire has been removed*. It seems that more extensive tests are needed before the practical value of this method can be ascertained.

3 Shock wave in presence of boundary layer

3.1 Shock wave and boundary layer interaction

As already remarked, if diffusion is to be achieved by means of a contracting duct with an efficiency higher than that of a normal shock at the free stream velocity, a shock must occur inside the diffuser, in the presence of the boundary layer. This is also true for the diffusers used in the supersonic wind tunnels and for annular entry, pitot type diffusers, such as shown in fig.5.

The normal shock in the presence of the boundary layer no longer conforms to the simple one-dimensional theory. The observations

* A 35 mm. film has been made of these tests at L.F.A., Völkenrode.

of shocks in supercritical flow around aerofoils and in supersonic nozzles suggest roughly the following mechanism of boundary layer and shock wave interaction.

Due to an adverse pressure gradient, the flow in the subsonic part of the boundary layer is reversed in the vicinity of the shock. The boundary layer thickens upstream of the shock and thus produces a number of oblique shock waves, such as are observed in high speed flow around aerofoils. The thicker the boundary layer and the greater the wetted perimeter of the stream, the more pronounced these effects become. In other words, for a given stream velocity, the effect of the boundary layer would depend on the Reynolds Number and would diminish with the increase of the Reynolds Number. On the other hand, for a given Reynolds Number, the intensity of shocks increases with velocity, so that the effect of a given boundary layer would be expected to increase with increasing Mach Number.

From the above interpretation of shock wave and boundary layer interaction the so-called "softening" of normal shocks by a series of oblique shocks might be expected to occur, the velocity in front of the normal shock being appreciably reduced and the overall losses correspondingly decreased. Although this may be true in certain conditions, additional losses are usually present.

It appears that the mechanism of the shock wave and boundary layer interaction has been first described on the above lines by Donaldson (ref.5), who also presented an experimental investigation of shock wave formation in a de Laval nozzle. A good estimate of the shock wave position was obtained by assuming that the shock occurs at a Mach Number equal to unity (i.e. a completely softened shock) and that the flow is isentropic throughout (supersonic upstream of the shock and subsonic - downstream). This result seems to be confirmed by investigations of flow around airfoils at supercritical speeds (quoted in ref.5), in which it was found that the losses through the shock near the airfoil surface may be smaller than those corresponding to the one-dimensional theory. It was also found that the Mach Number behind the base of a normal shock on an airfoil was close to unity. This was also observed in the entry tests mentioned later (ref.13), in which a Mach Number of 1.18 to 0.935 was observed in the vicinity of the shock, for a free-stream velocity of M = 1.46. Further, the occurrence of a few normal shocks in succession, e.g. in a de Laval nozzle, ref.5, can be explained by assuming a compression to sonic velocity through each "shock" followed by a renewed supersonic expansion.

Although, as already remarked, the losses in certain regions of softened shocks may be smaller than those for corresponding normal shocks, it appears that in general the total losses exceed those predicted by the one-dimensional theory.

A number of experimental results is available in this connection and will be reviewed here. The results were obtained mostly from performance data of diffusers for supersonic wind tunnels, but a few tests of pitot type entries in which the shock occurs in the presence of the boundary layer are also available.

3.2 Annular entry pitot type diffuser

The design of a supersonic entry or diffuser of the type shown in fig.5 is advantageous, for constructional reasons (e.g. accommodation for the pilot), for a projected supersonic aircraft. This so-called annular entry is essentially of the simple pitot type described before, but it differs in that the flow is deflected in front of the entry plane by the tip of the central body and the shock at entry occurs in the presence of the boundary layer formed on the central body surface.

Complete small scale models of diffusers of this type were tested at N.P.L. (ref.8) and tests of an analogous two-dimensional entry were carried out by Power Jets (R & D), Ltd. (ref.13). In all cases a much lower efficiency than for the simple pitot entries was recorded. In the N.P.L. tests the maximum flow obtained was only 0.91 of the flow through an area equal to the annulus in the free stream. Further, in both sets of tests the flow was stable only when shock occurred inside the diffuser, downstream of the entry ("overexpanded" condition). Surging started when the shock formed at the entry and was due to the softening effect of the boundary layer. In the Power Jets' tests attempts were made to remove the boundary layer at entry by suction, using either flush or projecting slits. It appears however that an effective control can be achieved only by passing the whole of the boundary layer through a projecting slit, thus making an entry of the simple pitot type.

It should be mentioned that the above tests were carried out at small Reynolds Numbers, the boundary layer effects being thus exaggerated. In the N.P.L. tests the Reynolds Number based on the distance from the nose of the model to the annular lip was equal to 6×10^5. In all tests the Mach Number did not exceed 2.0.

As pointed out by Lean (ref.8), apart from low efficiency and low maximum flow, the application of the annular entries would be difficult in practice because of their surging characteristics. In order to avoid the reversal of flow when the engine demand is low, it would be necessary to by-pass some of the air, the annulus being however large enough to pass the maximum flow required. Alternatively, in order to avoid the extra drag, the entry area could be varied; this, of course, would introduce mechanical complications.* Provision of a slit sufficient to remove the whole of the boundary layer would also increase drag.

Such complications are largely eliminated when simple pitot entries are used. At engine demands higher than the "swept volume" diffuser flow the compression efficiency would decrease due to the overexpansion (shock inside the diffuser) and the engine demand would adjust itself to the intake supply. At lower engine demands a stable, detached curved shock front would form ahead of the diffuser entry.

3.3 Diffusers for supersonic wind tunnels

Further experimental evidence of the effect of boundary layer on shocks comes mainly from tests of diffusers for supersonic wind tunnels.

(i) Diffusers without contraction

Castagna (ref.3) carried out an extensive investigation of supersonic flow in long, divergent conical ducts (6° vertex angle) with shocks occuring at various cross-sections and Mach Numbers ranging from 1.2 to 4.5. His results are shown in fig.6 in terms of the isentropic efficiency and the Mach Number immediately in front of the shock, as estimated from the observed pressure distribution and the initial state of the gas. In this, as in the other cases here considered, it was not possible to separate the losses due to the shock from those occurring downstream in the subsonic diffusion. Accordingly the efficiency is based on the observed static pressure ratio: (exit pressure/ lowest recorded pressure in the duct) and equation (A1), which assumes a compression to zero velocity. As in Castagna's tests the exit velocity did not exceed 0.3M, such an assumption is permissible.

In the same figure results of similar tests (ref.17) made at N.P.L. with a two-dimensional diffuser (5^o total divergence angle) for a supersonic wind tunnel are shown, the efficiency being based on the Mach Number in front of the diffuser, i.e. in the working section. Some early results obtained by Stodola and Steichen, presumably with similar diffusers, and quoted by Crocco (ref.4) are also included.

Castagna's and N.P.L. tests show a fair agreement (fig.6), but give efficiencies considerably lower than the theoretical ones for a normal shock and lossless subsonic diffusion, curve 1.

(ii) Diffusers with contraction

Another set of results, fig.7, refers to diffusers in which a second throat was employed, with shock occuring close to the contraction on its downstream side, i.e. at a velocity smaller than the maximum one at the diffuser entry. The maximum theoretical efficiency of such diffusers for the limiting contraction ψ_{min} has been calculated and is shown in fig.9, curve 4, also table II, page 14. It exceeds only slightly the normal shock efficiency, the difference being even smaller in practice.

In fig.7 are shown the results obtained by Ackeret and Brown Boveri Co., (ref.15) from small scale tests prior to the construction of the supersonic wind tunnel at Zurich*. Two sets of points are shown, referring to the diffuser only and to the compressor, the latter including losses in the whole system (nozzle, ducting, etc.). The N.P.L. tests of a two-dimensional diffuser for a 1.5 x 1.5 in. model supersonic wind tunnel (ref.17) show a fair agreement with the Swiss results, except at low Mach Numbers, where the efficiency drops suddenly. A similar behaviour was also observed at N.P.L. for other diffusers, but no adequate explanation has been given. A tendency for the efficiency curve to attain a maximum at low velocities would be expected when losses in the subsonic diffusion are taken into account (cf. fig.9). In the N.P.L. diffuser in question, the contraction ratio was 0.906 and the total angle of divergence which followed was 9^o.

A third set of results for continuous contraction diffusers represents the operating characteristics of a German 40 x 40 cm. supersonic tunnel at Wasserbau Versuchs Anstalt, Kochel (ref.11). The tunnel is of an open jet, intermittent flow type and a variable diffuser throat is used to adjust the pressure outside the jet to the value of the jet static pressure at the nozzle exit. The efficiency shown takes into account the losses upstream of the diffuser and is slightly lower as compared with the Swiss and N.P.L. results, no doubt chiefly because of the use of the open jet. The pressure drop across the silica gel driers is also included in the efficiency, its effect being however negligible, especially at high Mach Numbers (i.e. small flows); for $M > 2.0$ the drier pressure drop is smaller than 13 mm. Hg. In the subsonic part, the diffuser was conical of an 8^o vertex angle.

Comparing fig.6 and 7 it is not possible to establish any definite difference between the two types of diffuser.

(iii) Bridge type diffusers

A third type of supersonic diffuser for use in supersonic wind tunnels has been developed in the U.S.A. and in Great Britain during

* It appears that the same tests were quoted by Crocco, ref.4.

the war (ref.14 and 17) and is known as the bridge type diffuser. It consists of the usual divergent duct with a wedge-shaped bridge inserted across the entry section, with or without contraction around the bridge. Such a design is convenient from the point of view of the experimental technique, the bridge supporting a pressure traverse or model, and may also produce a more stable and efficient system of shock waves than that obtained with the diffusers hitherto described. The obvious fundamental requirement of the bridge, which in practice is only satisfied at higher velocities, is that its leading edge angle should not exceed the maximum value at which the shock wave becomes detached.

In fig.8, curve 2, are shown results obtained at N.P.L. (ref. 17) with the contracting diffuser for the 1.5 x 1.5 in. supersonic tunnel already mentioned but with a 0.12 in. thick bridge inserted at the beginning of the diffuser contraction. It appears that at high velocities the bridge type gives a somewhat better efficiency. Other N.P.L. tests (ref.6), obtained presumably with bridge type diffusers as finally used in the N.P.L. 11 x 11 in. tunnel, also show a relatively high efficiency (curve 3).

Bridge type diffusers were also tested in the U.S.A. in connection with the design of a 2.50 x 2.56 in. model supersonic wind tunnel (ref.14). Only two test points are available and, as shown in fig.8, they are in good agreement with the N.P.L. results. The walls of the diffuser were shaped so as to maintain a constant area cross-section around the wedge.

Other N.P.L. tests (ref.17) indicate that a sudden expansion round the bridge causes a fall of efficiency.

(iv) Discussion of data on wind tunnel diffusers

From the results reviewed, it is evident that the efficiency of diffusers as used at present in supersonic wind tunnels is, in general, low. There are several factors, however, to account for this apparent neglect in the tunnel diffuser design.

In the first instance, the velocities attained in the majority of the existing supersonic tunnels do not exceed $M = 5.0$ and the working sections are relatively small, so that, even with low diffuser efficiencies, the power requirements can be easily met. Further, since usually one universal diffuser operates over the whole range of velocities, it is impracticable to attempt a more efficient design, e.g. on the lines described in the next section. The non-uniformity of flow behind the model makes such an attempt in any case hardly possible. It is therefore understandable that the use of a variable throat or bridge type diffusers should have been dictated rather by the exigencies of the experimental technique than by efficiency considerations. It seems that the present methods of diffuser design may have to be revised in case of tunnels of very high Mach Numbers and large cross-section.

4 Multi-shock diffusers

4.1 Introduction to the problem

In the previous sections the impracticability of a diffusion from supersonic velocity without loss, by means of a reversed nozzle, has been demonstrated. In practice, when diffusers of this type are used, the compression occurs through a normal shock at the free stream, or, when the limiting contraction ψ_{min} is used, at a slightly lower velocity. The corresponding theoretical efficiencies and pressure recoveries are shown in fig.9 and 10 and tabulated in table II.

Table II (γ = 1.400)

Calculated efficiency of single-shock supersonic diffusers.

M	η_σ of normal shock for $\eta_{\sigma\,sub}$ equal to:				η_σ of diffuser with limiting contraction ψ_{min} and $\eta_{\sigma sub}$ = 1.0
	1.0	0.7	0.5	0	
1.0	1.00	0.70	0.50	0.00	1.00
1.2	0.99	0.82	0.71	0.44	-
1.4	0.96	0.85	0.78	0.61	-
1.6	0.91	0.84	0.79	0.67	0.96
2.0	0.80	0.76	0.73	0.67	0.87
2.5	0.68	0.65	0.64	0.60	0.75
3.0	0.58	0.56	0.55	0.53	0.64
3.5	0.50	0.49	0.48	0.46	0.55
4.0	0.43	0.43	0.42	0.40	0.49
4.5	0.38	0.38	0.37	0.36	0.43
5.0	0.34	0.34	0.33	0.32	0.38

For the normal shock, curves for various subsonic efficiencies smaller than 1.0 are also drawn. In particular, the following expressions are derived for normal shock with subsonic efficiencies of unity and zero:

for $\eta_{\sigma\,sub}$ = 1.0,

$$\eta_\sigma = \frac{\gamma + 1}{\gamma - 1}\left\{\frac{\gamma + 1}{2\gamma M^2 - (\gamma - 1)}\right\}^{\frac{1}{\gamma}} - \frac{2}{(\gamma - 1)M^2} \qquad (5)$$

and for $\eta_{\sigma\,sub}$ = 0,

$$\eta_\sigma = \left\{\left[\frac{2\gamma M^2 - (\gamma - 1)}{\gamma + 1}\right]^{\frac{\gamma-1}{\gamma}} - 1\right\}\Big/\left(\frac{\gamma - 1}{2} M^2\right) \qquad (6)$$

The experimental results reviewed have shown that the calculated theoretical efficiencies can be attained only when the shock is not affected by the presence of the boundary layer.

In fig. 9 and 10 two other curves are drawn for oblique and conical shocks, with isentropic subsonic compression. Curve 2 applies to both the oblique and conical shocks when velocity behind the shock is sonic. Curve 3 shows a slightly higher efficiency, the sonic velocity being reached at the cone surface. In both cases the efficiencies and pressure recoveries are substantially the same as for the reversed nozzle with the limiting contraction ψ_{min} (curve 4).

On the whole, fig. 9 and 10 show that at velocities exceeding say M ≈ 2.5 the single-shock diffusers have a small compression efficiency, which tends to zero for infinite Mach Numbers, so that other means must be sought to achieve an efficient diffusion at very high speeds.

Apart from the reversed nozzle flow, an isentropic compression from the supersonic velocity can be theoretically achieved through a deflection of the flow by means of a suitably curved surface. In two-dimensional flow the correct surface profile is determined by a streamline of the well known Prandtl-Meyer solution, whereas in the axially symmetrical flow the shape of the body of revolution is obtained by a modified method of characteristics. We shall limit ourselves to examples of the two-dimensional solutions but the conclusions drawn will also apply to the axially symmetrical flow, provided the profiles of the deflecting surfaces are suitably modified.

Experimental data on compression of the above type is, at present, very limited. Ackeret (ref.1) presented a Schlieren photograph of two-dimensional, supersonic flow, compressed by deflection, but it appears that the only tests of complete diffusors were made in Germany towards the end of the war, chiefly by Oswatitsch, who used axially-symmetrical models. These experiments, to which reference will be made later in greater detail, have shown that in practice a lossless compression of the above type cannot be obtained because of the boundary layer influence. The boundary layer tends to form "dead water" regions and the compression always occurs through a number of finite (oblique or conical) shocks, the overall efficiency being nevertheless much higher than that of a normal shock. This effect is similar to the softening of normal shocks already described; in fact the latter phenomenon indicates the way in which an efficient compression can be realized.

Thus in practice, instead of an isentropic compression, one is confronted with a multi-shock compression. The theoretical aspects of the design of multi-shock diffusors and the experimental results available will be now considered. It is convenient to discuss the former under two separate headings, covering the geometry of design and the efficiency of compression.

4.2 Geometry of design

Although the compression occurs through a number of finite shocks, it is convenient to discuss the basic principles of the geometrical lay-out of multi-shock diffusors by inspection of the isentropic flow patterns; the conclusions reached are quite general.

In fig.11 several ways, in which a compression from $M = 1.918$ down to sonic velocity can be achieved, are illustrated, the Mach lines being drawn for 4° intervals of flow deflection. Although such diffuser arrangements as shown in fig.11 could be used, usually a symmetrical design will be preferred, a central body thus being formed (similar to fig.5).

A supersonic flow along any concave or convex surface is represented by drawing a. When a correct concave shape is not used, the Mach waves overlap one another and a curved shock front develops at some distance from the surface. To avoid this, a focussed wave can be used, as shown in b. This is, for the two-dimensional case, the reversed Prandtl-Meyer expansion round the corner. The profile of the curved surface corresponds to a streamline and the sharp corner forms the outer diffuser rim.

Starting with the flow pattern b, several arrangements can be developed (c to f). Diffusors d and f are analogous to c and e, in which however a focussed wave system is used. The advantage of a focussed system is clearly demonstrated: the "supersonic length" l and the dimension R, which determines the minimum diffuser width or diameter, are both considerably smaller than for non-focussed waves.

When the flow is deflected in one direction only (through a focussed wave), as in c, the dimension R has the smallest possible value and is equal to the free-stream width or radius; in all other cases R exceeds this minimum value.

The criterion for the design of the outer diffuser rim is easily defined (cf. ref. 2). For both two-dimensional and axially symmetrical diffusers the flow past the rim can be regarded as two-dimensional. In order to prevent the flow on the inside of the rim from being disturbed, the angle δ, which the outside tangent at the rim edge makes with the local direction of flow must be smaller than the critical value, at which the shock becomes detached. These critical values of δ are shown in terms of the Mach Number in fig. 13 (for $\gamma = 1.4$). The critical angle δ increases with velocity and will be therefore theoretically determined for the lowest velocity at which the diffuser is required to operate.

The above criterion has been verified experimentally for simple pitot entries (ref. 16). It was found that the outside rim angle δ had no influence on flow inside the diffuser, provided it did not exceed the critical value.

In course of the compression of the type considered the flow is deflected before reaching the sonic velocity. For a two-dimensional, isentropic case, the angle of deflection ν is given in fig. 13 (for $\gamma = 1.4$). With oblique shocks, the flow deflection is, for a single shock, only a little smaller than the critical wedge angle δ and, as the number of shocks increases, it approaches the isentropic angle ν.

The deflection of flow gives the correct theoretical inclination of the inside rim surface. This condition, however, can be only satisfied if the critical angle δ is greater than the deflection angle ν. From fig. 13 it is evident that in the case of a single focussed wave (c), $\nu > \delta$ and the flow must be abruptly deflected inside the rim, if the correct value of δ is used. Alternatively, with a correct value of ν (cf. fig. 12, top), the shock becomes detached from the rim.

In order to reduce the overall flow deflection, designs of the type c and f can be used. Here the waves are split in groups which deflect the flow in opposite directions, thus reducing (in the examples shown - down to zero) the total deflection. Again, a better and more compact design is obtained with the focussed waves (type c). It seems that for very high velocity diffusers of this type are suitable.

Flow patterns analogous to c and e are shown, for oblique shocks, in fig. 12. In both cases two oblique shocks are followed by a normal one. The advantage of a reversed deflection is well demonstrated.

The absolute minimum size of the diffuser entry is that of the free-stream cross-section corresponding to the diffuser flow. With focussed waves this minimum size has to be only slightly exceeded, as shown by cases c and e. The diffusion will usually be required to proceed to quite a low subsonic velocity and the question of the necessary cross-sectional area arises. From fig. 3 it is seen that for isentropic flow the entry (free-stream) cross-section is quite adequate to achieve a diffusion to low Mach Number, provided the free stream Mach Number is sufficiently high; the same holds when losses occur. It thus seems that only for diffusers operating at low entry Mach Numbers it may be necessary to exceed, in the subsonic region, the entry cross-sectional area.

4.3 Efficiency of compression

The geometrical principles which should be observed in multi-shock diffuser design were considered above and it remains now to analyse the thermodynamic efficiency of this type of diffuser.

In the simplest case, the compression would occur through one, oblique or conical, shock with an efficiency, as already pointed out, only slightly higher than that of a normal shock (cf. fig.9 and 10).

When the number of shocks is greater than one, it is possible to find, for a given set of conditions, an optimum arrangement of shocks for maximum overall efficiency. Such analysis will not be undertaken here, but the efficiency of multi-shock diffusers will be investigated for certain simple design criteria. The results, fig.14 to 18, are given in terms of the free stream Mach Number M and the ratio p_σ of the isentropic stagnation pressures after and before the diffusion (cf. (A2)). An isentropic subsonic compression is assumed. By means of fig.21 or 22 the corresponding efficiencies η_σ can be determined. The results given in fig.14 to 18 were obtained by graphical methods and must be regarded as approximate only.

Two different design criteria were used: either the entropy rise $\Delta\sigma'$ across each oblique shock or else the angle of flow deflection δ' through each shock were kept constant, for each set of shocks considered. In each case three sets of curves, for various velocities after the oblique shocks (M = 1.0, 1.6 and 2.0), were obtained. In each figure lines of constant number n of oblique shocks are drawn, together with the lines of constant entropy increase $\Delta\sigma' = -\log_e p_\sigma'$ or constant angle of deflection δ', across each shock, as the case may be. If the velocity after the series of oblique shocks is supersonic, a normal shock is assumed to occur (i.e. at M = 1.6 and 2.0). Provided the normal shock occurs at a sufficiently low velocity, it does not affect adversely the overall efficiency. On the other hand, the presence of the normal shock may be beneficial in practice in that it provides a certain amount of flexibility in the diffuser operation: the final pressure depends on the position of the normal shock in the divergent subsonic part of the diffuser (cf. fig.12).

It will be noticed from fig.14 to 17 that, since the families of curves are drawn for constant velocities before the normal shock, the lines of each family start from a point corresponding to this velocity on the p_σ - M curve for the normal shock.

Whichever criterion is used, for a given velocity M, the efficiency of compression increases with the number of shocks. By superimposing the curves for the same number of shocks but for different velocities before the normal shock, envelope curves of an optimum pressure recovery, for a given number n of oblique shocks followed by a normal one, are obtained; they are shown in fig.18 for shocks of constant entropy increase $\Delta\sigma'$ and constant deflection angle δ'. It appears that in the former case the maximum pressure recovery, for a given n, is greater, the differences being however small.

From fig.18 it is evident that by suitably increasing the number of oblique shocks with the free-stream velocity, very high isentropic efficiencies can be obtained, e.g. for M = 30 and two oblique shocks, $\eta_\sigma \approx 0.88$m whereas for M = 4.0 and 3 oblique shocks, $\eta_\sigma \approx 0.84$. Further, for an optimum efficiency, the velocity at which the normal shock should occur increases for a given n, with the free-stream Mach Number.

The theoretical efficiencies appear to be of the same order for the axially-symmetrical diffusers. The calculations carried out in connection with tests of diffusers of this type (ref.10) give, for 3 conical shocks of constant entropy rise and one normal shock at $M = 1.5$, a theoretical pressure recovery $p_r \approx 0.80$ or $\eta_r \approx 0.90$ at $M = 2.9$ (see fig.19, 20), which is only slightly smaller than the corresponding value for the two-dimensional case (cf. fig.15).

From the above considerations it is apparent that theoretically any required compression efficiency can be obtained provided the number of shocks is sufficiently large; in fact, in the limit, for $n = \infty$ and sonic velocity after the last shock, the compression becomes isentropic. As already observed, in practice the compression tends to occur through a finite number of shocks, but the effects of the boundary layer are at present too little known to allow of the determination, for any given set of conditions, of the maximum number of shocks actually possible and the corresponding maximum efficiency attainable.

4.4 Experimental results

The practicability of multi-shock diffusers has been demonstrated by a series of tests carried out in Germany by Oswatitsch (ref.10) and others. The available information on these tests is at present incomplete and only some of the results can be reviewed.

As stated by Oswatitsch, the performance of the two-dimensional, multi-shock diffusers, with which a few tests were made, was found unsatisfactory. The main set of tests was done at Göttingen using axially-symmetrical models designed for a continuous compression, or two or three inclined shocks* followed by a normal shock. Various shock wave patterns and outer rim designs were tested. As already mentioned, it was not found possible to achieve a continuous compression, but tests of multi-shock diffusers have shown a good agreement with the theory.

In ref.10 tests of axially-symmetrical diffusers designed for 3 inclined and one normal shock are described. According to a statement made by Oswatitsch when interviewed, but not included in this report (ref.10), the entropy rise across the inclined shocks was kept constant and the normal shock was to occur, at the point of the highest efficiency, at $M = 1.5$.

The above report does not give in detail the shape and dimensions of the tested diffusers or the corresponding calculations, but contains only their general description and main experimental results. Of the three diffusers tested one was designed to give a maximum compression efficiency and another to give an optimum overall performance; the tests of these diffusers will here be briefly summarized.

In fig.19 and 20, reproduced from ref.10, the two diffusers, No.III and VI, together with their pressure recovery characteristics are shown. Diffuser No.III was designed to give a maximum compression efficiency and a maximum mass flow. A shock wave formation, focussed somewhat in front of the outer rim, was used (analogous to case c, fig.11) and the smallest cross-section was located at entry.

* In the case of axially-symmetrical diffusers only the first shock, originating from the tip of the diffuser, occurs in a uniform velocity field and is truly conical; subsequent inclined shocks form curved surfaces.

In order to reduce the inevitably high drag of the above design, diffuser No.VI was constructed so that the flow was deflected through the third shock in a direction opposite to that of the first two deflections (analogous to case e, fig.11); the smallest cross-section occurred inside the diffuser.

At zero incidence and at the correct design Mach No. $M = 2.9$, the measured pressure recovery p_r attains, for various positions of the normal shock, practically the corresponding theoretical values shown in fig.19 and 20; the same agreement was found for the mass-flow, which was smaller by only a few per cent than the theoretical value (see table III) and remained constant for all positions of the normal shock beyond the critical one. The position of the normal shock was adjusted by altering the exit area of the diffuser F_D. It was found that as F_D was reduced an unstable critical condition, similar to that mentioned in the case of an annular pitot entry, developed. Instability commenced at larger exit areas, and consequently with a lower maximum pressure recovery, than is theoretically possible. The flow became unstable when the ratio

$$\frac{\text{diffuser exit area}}{\text{annular entry area}} = \frac{F_D}{F_e}$$

was smaller than about 1.1, whereas theoretically it should have been equal to about 1.0. For area ratios smaller than 1.1 the mass flow and the pressure recovery both decreased abruptly. It is probable that at higher Reynolds Numbers the unstable condition would occur nearer to the theoretical critical point.

Other characteristics of the two diffusers are compared in table III. Diffuser No.VI shows a reduction in the drag coefficient of 39% for a loss in efficiency of 3% only.

The effect of an incidence of 5° approx. was investigated at the design velocity and was found to be negligible, on the efficiency (cf. fig.19, 20) and mass flow. The drag and lift were investigated over a range from 0° to 8°, at the correct Mach Number of 2.9; the corresponding results are given in table III.

Table III

Characteristics of Oswatitsch diffusers.

				Diffuser No.III (fig.19)	Diffuser No.VI (fig.20)
At design Mach Number = 2.9	zero incidence $\alpha = 0°$	Maximum pressure recovery p_r	theoretical	≈ 0.80	≈ 0.80
			actual	0.705	0.65
		Maximum isentropic efficiency η_r	theoretical	≈ 0.90	≈ 0.90
			actual	0.85	0.82
		Mass flow, % of theoretical		97.3	93
		Measured drag coefficient C_D		0.56	0.34
		Measured drag coefficient C_D at $\alpha = 8°$		0.67	0.56
		Lift-incidence curve slope $dC_L/d\alpha$ (α in degrees)		0.055	0.06

As stated by Oswatitsch, the pressure recovery and mass flow tests were made using models of 65 mm. outside diameter. For the experiments involving drag and lift measurements smaller models were used, which were found to be considerably less efficient, this being attributed to the lower Reynolds Number.

In general, the above experiments show that by using a geometrically suitable shock pattern a considerable reduction in the diffuser drag can be achieved with no appreciable effect on the efficiency.

5 Supersonic diffusers at a Mach Number other than the design one

So far our considerations were confined practically only to supersonic diffusers which operated at the correct design Mach Number. In actual applications they will usually be required to operate over a range of supersonic velocities and, in some cases, even over the transonic region. The problem of a satisfactory design which would comply with such requirements is extremely difficult and, even if the transonic range were omitted, cannot be easily solved in the supersonic region alone.

It is only in the case of a normal shock formed at a simple pitot-entry (cf. fig.4) that the shock formation does not change with the Mach Number, although of course the compression efficiency decreases rapidly. In all other cases, provided the flow through the diffuser is not restricted, the shock wave pattern in a diffuser of a given design is solely a function of the free-stream Mach Number. Thus in a multi-shock diffuser of type c, fig.11, the shocks would be steeper at a velocity lower than the correct design Mach Number and they would enter the outer rim at higher velocities, than the correct. At lower velocities there would be a tendency towards the formation of normal shocks and the flow through the diffuser would diminish; in any case the compression efficiency would decrease. The obvious remedy would be to arrange for the diffuser shape to be altered according to the Mach Number; such a proposition results however in serious mechanical complications.

One of the above-mentioned German axially-symmetrical diffusers was tested by the W.V.A., Kochel, at velocities below and above (up to $M = 4.38$) the design Mach Number of 2.9, (ref.12). These tests have shown a rapid fall in efficiency at Mach Numbers smaller than the design Mach Number; the effect was less pronounced at higher Mach Numbers. This would be expected if shocks focussed, at the design Mach Number, in front of the diffuser rim were used; at higher Mach Numbers they would still be efficient, their intersection approaching the diffuser rim.

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Attached:- Appendix I
Figs.1 - 23 = Drg. Nos. Gas 5561 - Gas 5581

Distribution:-

C.S.(A)
D.G.S.R.
D.G.T.D.
D.Eng.R.D.
D.D.G.S.R.
D.D.Eng.R.D.
A.D.S.R.
D.D.Eng.R.
D.D.T.E.
A.D./R.D.E.(R)
Supersonic Committee (20)
R.T.P./T.I.B. (110 + 1)
Turbine Estab. (Pyestock)
A.R.D. Fort Halstead
N.P.L. (Aero)
N.P.L. (Eng.)

Director
C.S.(A)
Aero. Dept.
Controlled Weapons Dept.
Library (2)
G.D./A
G.D./P
G.D./P.B.

Appendix

Efficiency of adiabatic compression.

A number of quantities are used to indicate the efficiency of adiabatic compression; such quantities will be defined here and the relationships between them established. The latter are useful in reducing the experimental data to a common basis.

As usual, a compression to zero velocity will be assumed and the gas assumed to be perfect and to have constant specific heats. Denoting by 1 and 2 the states of the gas at the diffuser entry and after the compression to zero velocity respectively, and putting

γ = ratio of specific heats, $= c_p/c_v$,

P = static pressure,

P_σ = pressure attained by isentropic compression to zero velocity from any initial velocity so that, at a given Mach Number M,

$$P_\sigma/P = \left(1 + \frac{\gamma - 1}{2} M^2\right)^{\frac{\gamma}{\gamma - 1}},$$

$p = P_1/P_2$,

the following ratios are defined:

(i) Isentropic efficiency of compression

$$\eta_\sigma = \frac{\text{Work of isentropic compression cycle from the entry pressure } P_1 \text{ to the actual final pressure } P_2}{\text{Kinetic energy available at the diffuser entry}}$$

$$= \frac{p^{-\left(\frac{\gamma-1}{\gamma}\right)} - 1}{\frac{\gamma - 1}{2} M_1^2} \qquad (A1)$$

(ii) Ratio of pressures attained by a compression to zero velocity in an actual and in an isentropic diffuser.

$$p_\sigma = P_2/P_{\sigma_1} \qquad (A2)$$

(iii) Dimensionless entropy increase in adiabatic compression to zero velocity

$$\Delta\sigma = -\log_e p_\sigma \qquad (A3)$$

where $d\sigma = J ds/R$ with s = entropy,
R = characteristic gas constant,
J = mechanical equivalent of heat,

(iv) Isentropic power factor (assuming an isentropic compression in the compressor followed by cooling at constant pressure)

$$E_\sigma = \frac{\text{Work of isentropic compression cycle through pressure ratio } p_\sigma}{\text{Kinetic energy available at the diffuser entry}}$$

$$= \left(p_\sigma^{-\frac{\Upsilon-1}{\Upsilon}} - 1\right)\left(\frac{2}{\Upsilon - 1}\,\frac{1}{M_1^2} + 1\right) \qquad (A4)$$

(v) **Polytropic compression efficiency**

$$\eta_n = -\frac{dP/\rho}{d\left(\frac{w^2}{2}\right)} = \text{const.} \qquad (A5)$$

where ρ = mass density
and w = velocity of flow.

Only the first of the above quantities represents in the true sense the efficiency of adiabatic compression. The other quantities are useful in experimental work and for design purposes.

The results of diffuser tests are usually obtained in terms of the pressure ratio p_σ, which is correlated to the isentropic compression efficiency η_σ by the following equation:

$$p_\sigma^{\frac{\Upsilon-1}{\Upsilon}} = \left\{\frac{\Upsilon - 1}{2} M_1^2\,\eta_\sigma + 1\right\} \Big/ \left\{\frac{\Upsilon - 1}{2} M_1^2 + 1\right\} \qquad (A6)$$

This is represented, for Υ = 1.400, in fig.21 and 22, in the $\eta_\sigma - M_1$ and $p_\sigma - M_1$ co-ordinates.

The $p_\sigma - p$ relationship is given by the expression for isentropic flow already mentioned:

$$p\,p_\sigma = \left(\frac{\Upsilon - 1}{2} M_1^2 + 1\right)^{-\frac{\Upsilon}{\Upsilon-1}} \qquad (A7)$$

In some cases the diffuser test results (ref.4) are given in terms of the polytropic efficiency η_n, as defined by (A5). This definition implies that at every stage of the compression a certain constant fraction η_n of the kinetic energy is used for the pressure build up. Theoretically, there is no reason to base the definition of the adiabatic compression efficiency on this assumption; in some instances, however, it may prove useful in that it allows the determination of the intermediate states of the gas between the diffuser entry and the end of the compression.

Using the adiabatic flow energy equation and equation of state it can be shown that the assumption (A5) gives a polytropic compression P/ρ^n = const. with

$$n = \Upsilon\eta_n \Big/ \left\{1 - \Upsilon(1 - \eta_n)\right\} \qquad (A8)$$

When η_n = 1.0, n = Υ and the compression is isentropic.

From (A5) the following functions of η_n are obtained:

$$1/p = (P_{\sigma_1}/P_1)^{\eta_n} = \left(\frac{\gamma - 1}{2} M_1^{\,2} + 1\right)^{\frac{\gamma}{\gamma-1}\eta_n} \tag{A9}$$

$$1/p_{\sigma} = \left(\frac{\gamma - 1}{2} M_1^{\,2} + 1\right)^{\frac{\gamma}{\gamma-1}(1-\eta_n)} \tag{A10}$$

Combining (A10) with (A6), the $\eta_{\sigma} - \eta_n$ relationship is obtained in terms of M_1

$$\left(\frac{\gamma - 1}{2} M_1^{\,2} + 1\right)^{\eta_n} = \frac{\gamma - 1}{2} M_1^{\,2}\,\eta_{\sigma} + 1 \tag{A11}$$

and is drawn, for $\gamma = 1.400$, in fig. 23.

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REPT. GAS 8
FIG. 1, 2

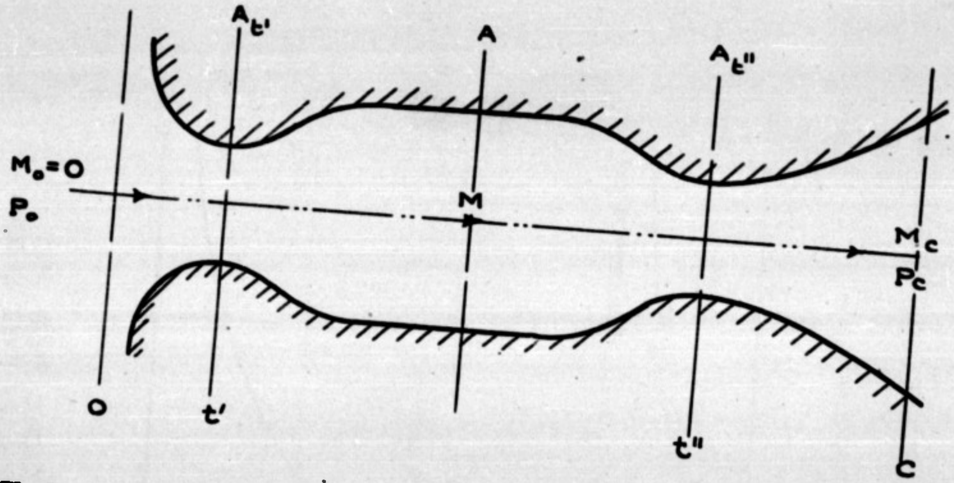


FIG. 1 TWO-THROAT FLOW SYSTEM

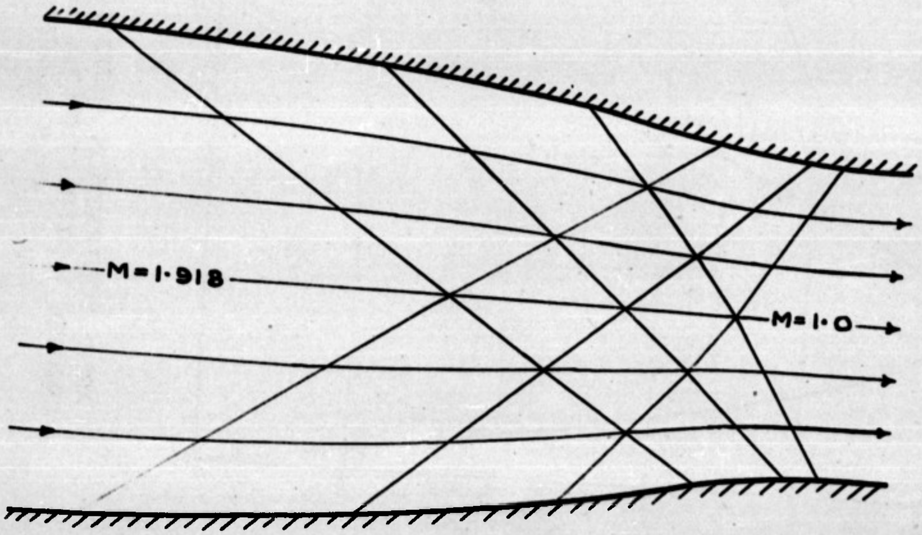
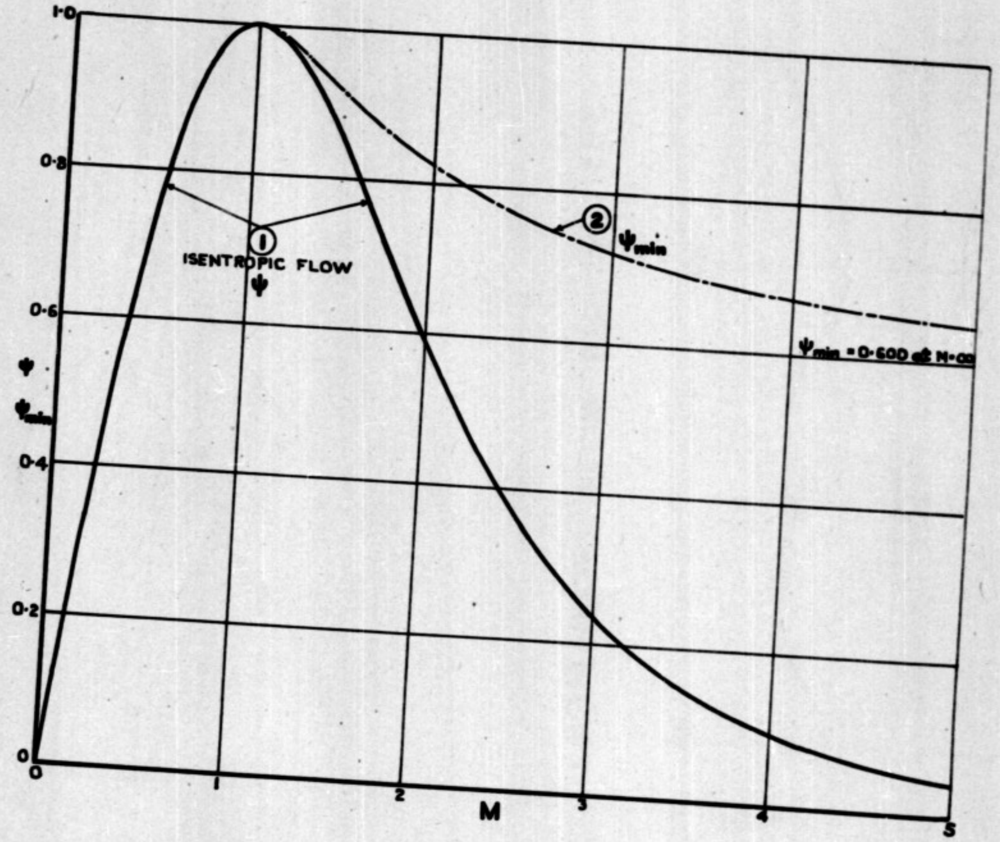


FIG. 2
TWO DIMENSIONAL, ISENTROPIC SUPERSONIC DIFFUSER (REVERSED NOZZLE)

20

FIG. 3.



AREA CONTRACTION RATIO ψ IN ISENTROPIC FLOW AND LIMITING CONTRACTION ψ_{min} IN FUNCTION OF FREE STREAM MACH NUMBER M (γ = 1·4000)

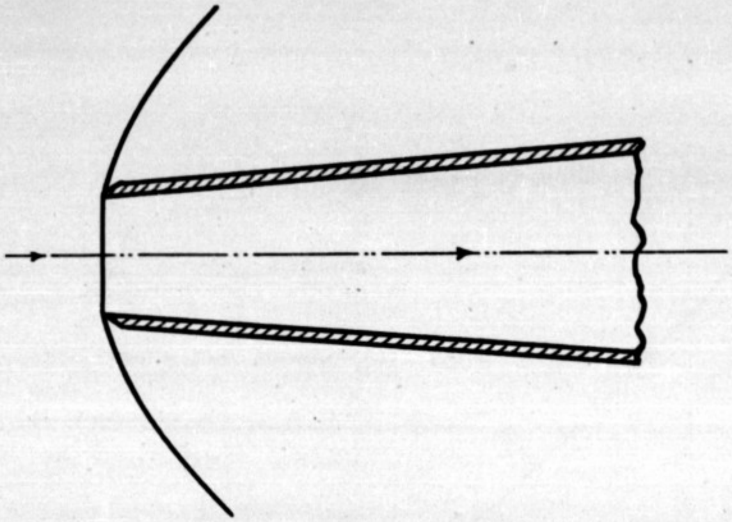


FIG. 4 SIMPLE PITOT TYPE DIFFUSER

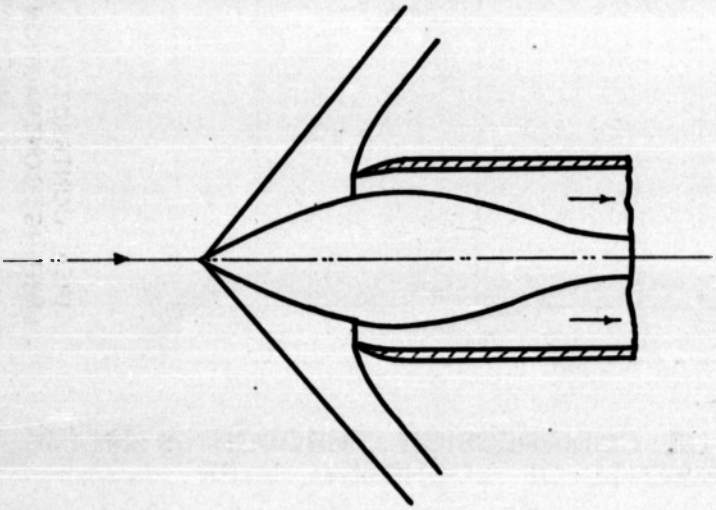
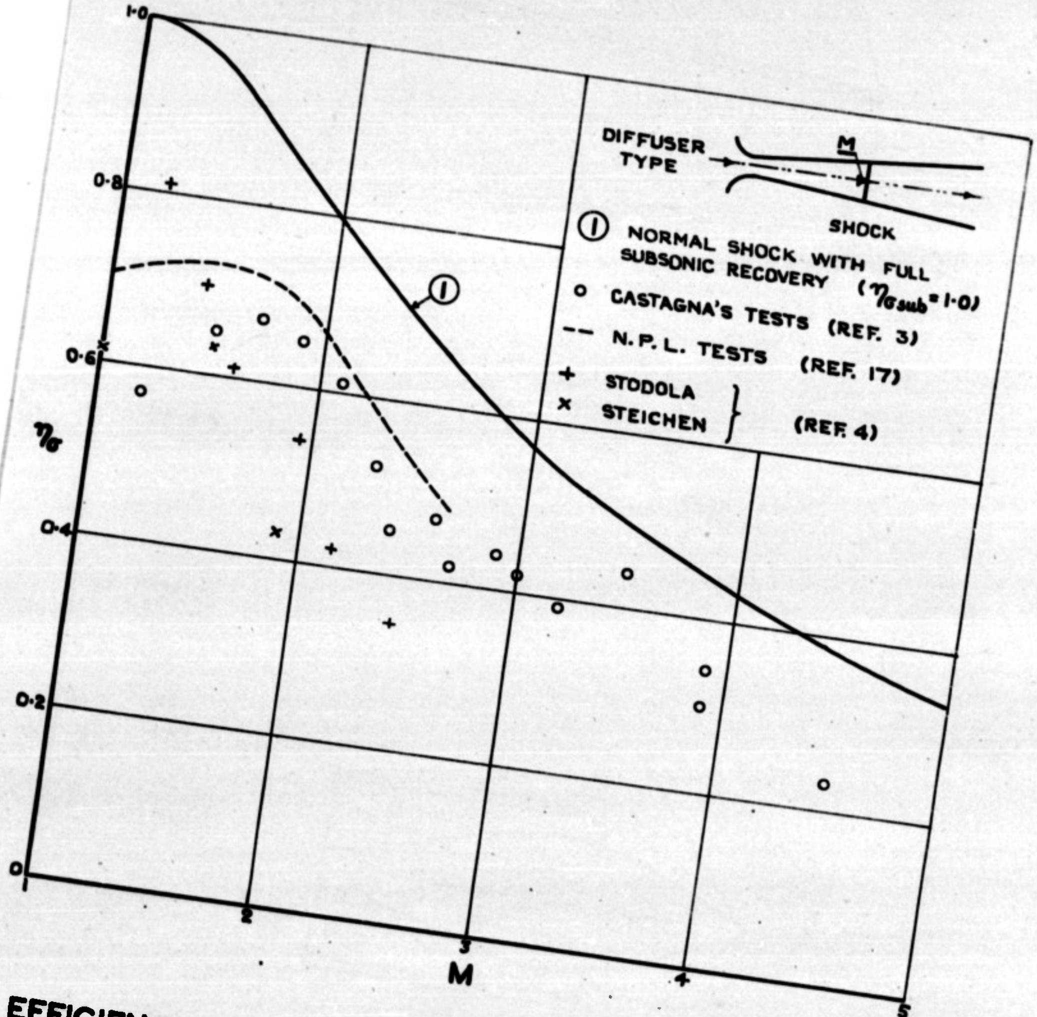


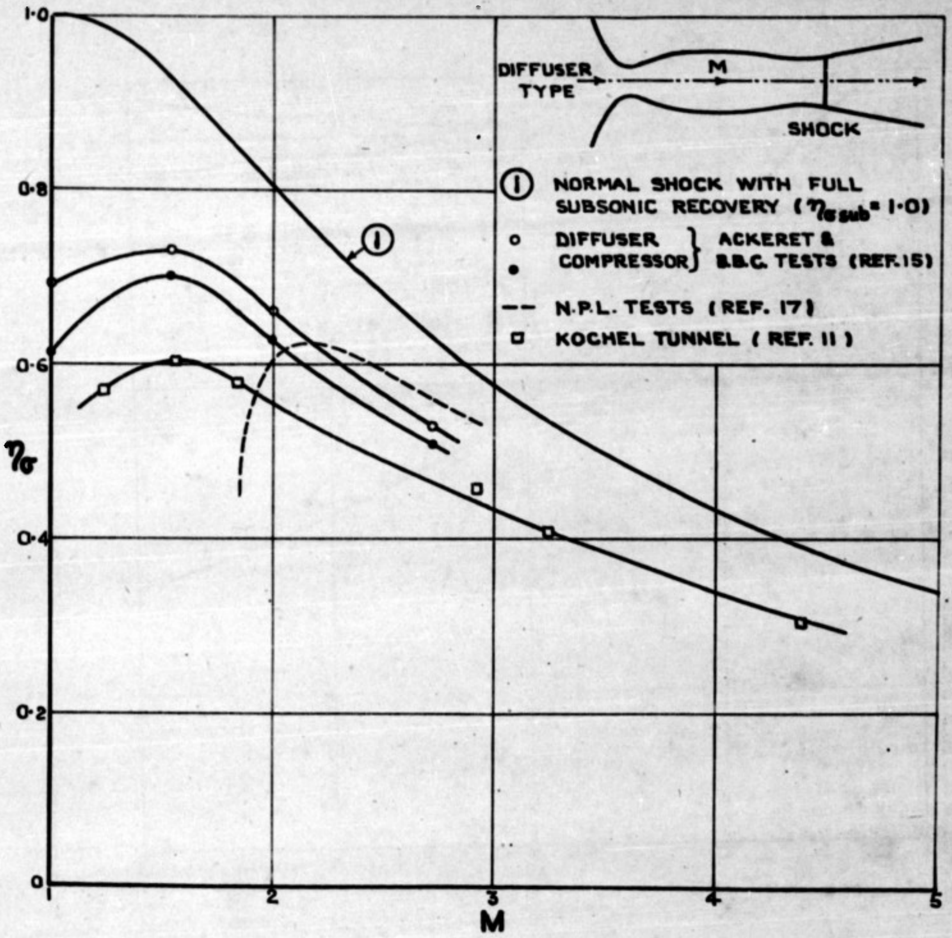
FIG. 5 ANNULAR ENTRY PITOT TYPE DIFFUSER



EFFICIENCY OF COMPRESSION THROUGH A SHOCK IN PRESENCE OF BOUNDARY LAYER

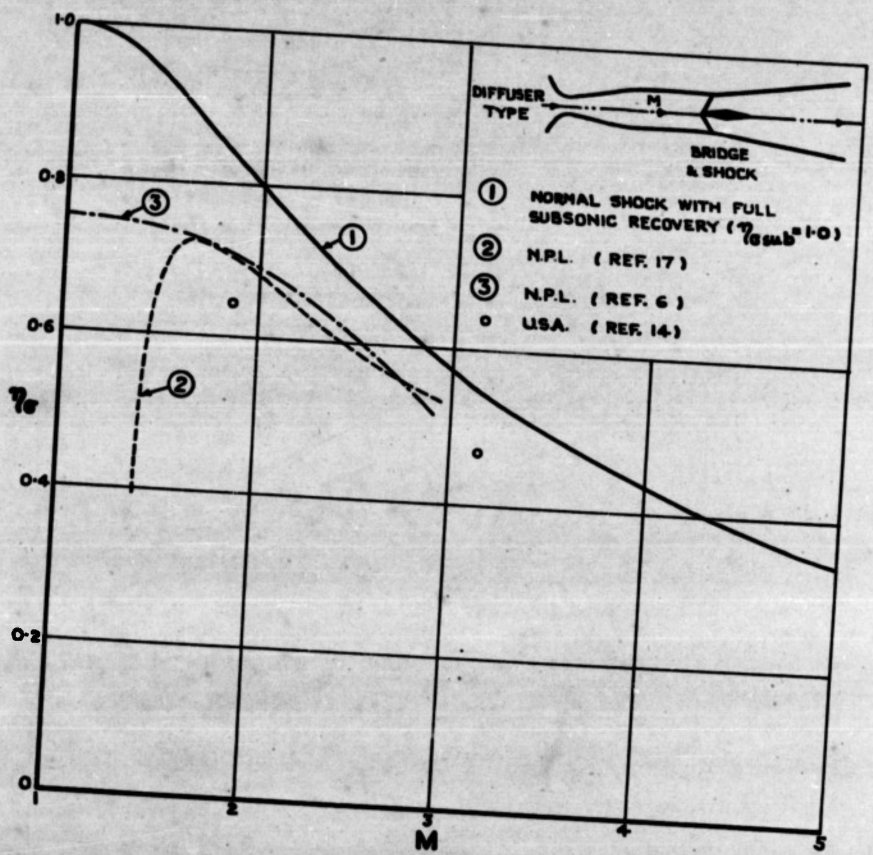
39

FIG. 7.



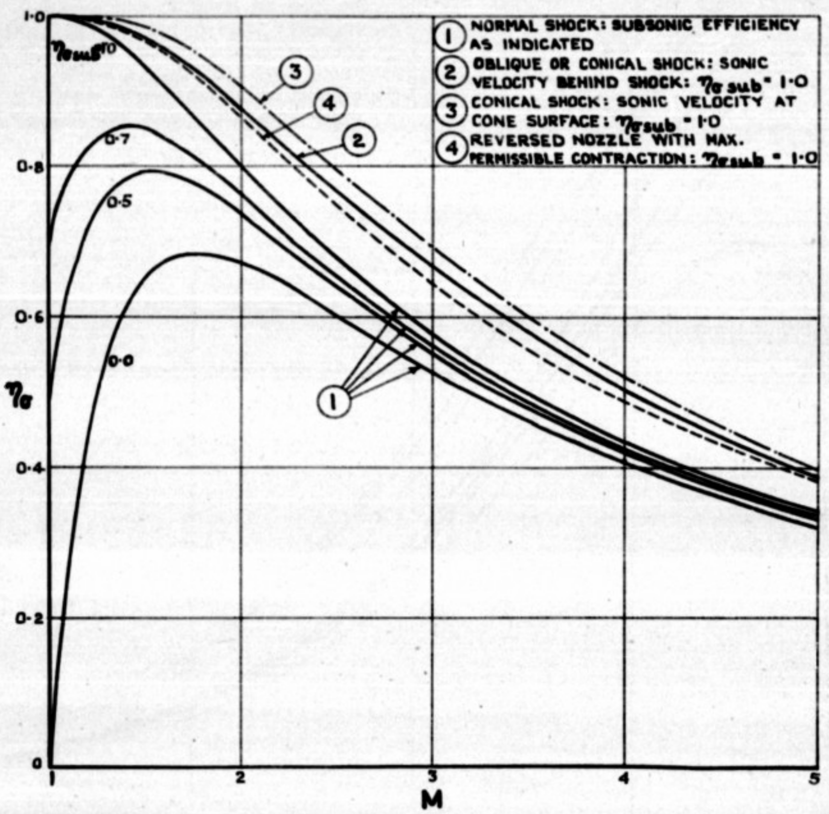
EFFICIENCY OF COMPRESSION THROUGH A SHOCK IN PRESENCE OF BOUNDARY LAYER

GAS. 5565.



EFFICIENCY OF COMPRESSION THROUGH A SHOCK IN PRESENCE OF BOUNDARY LAYER.

FIG. 9.

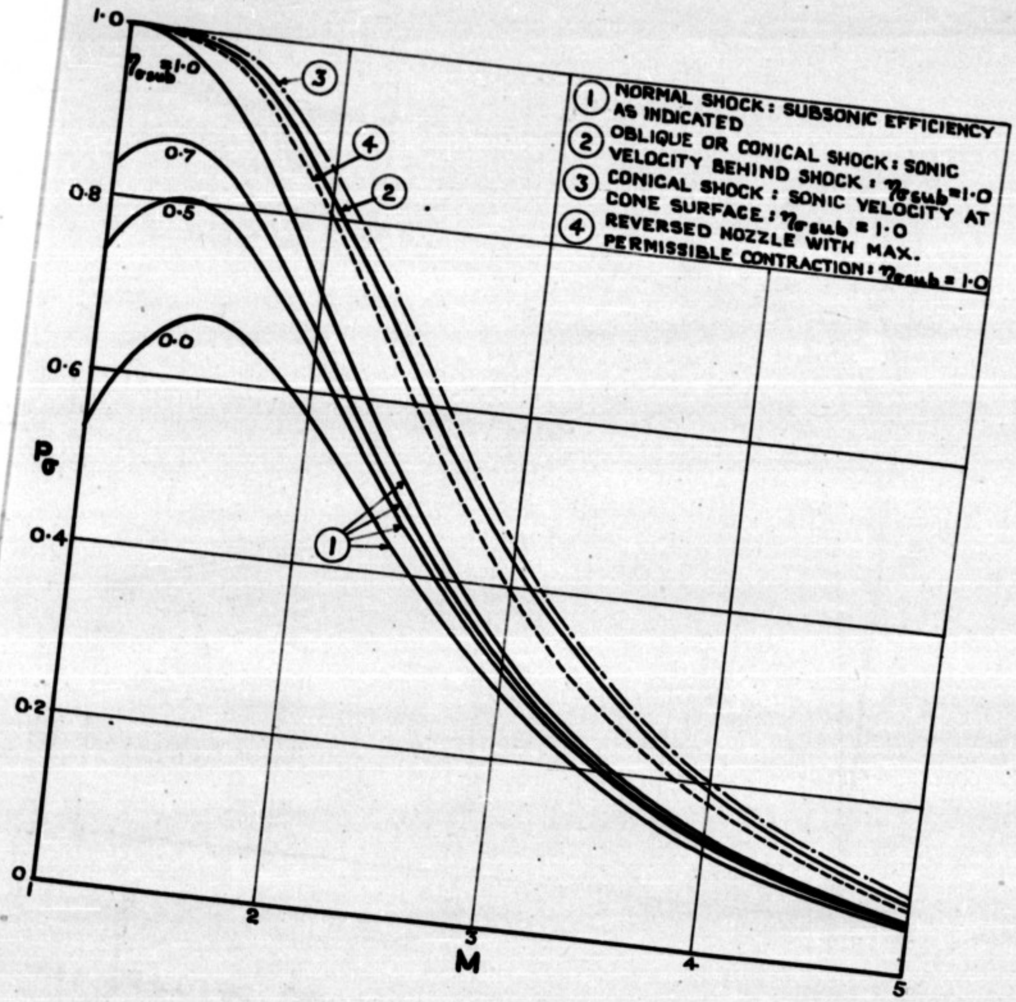


EFFICIENCY OF SINGLE-SHOCK SUPERSONIC DIFFUSERS (γ = 1·4)

GAS. 5568.

REPORT GAS. 8.

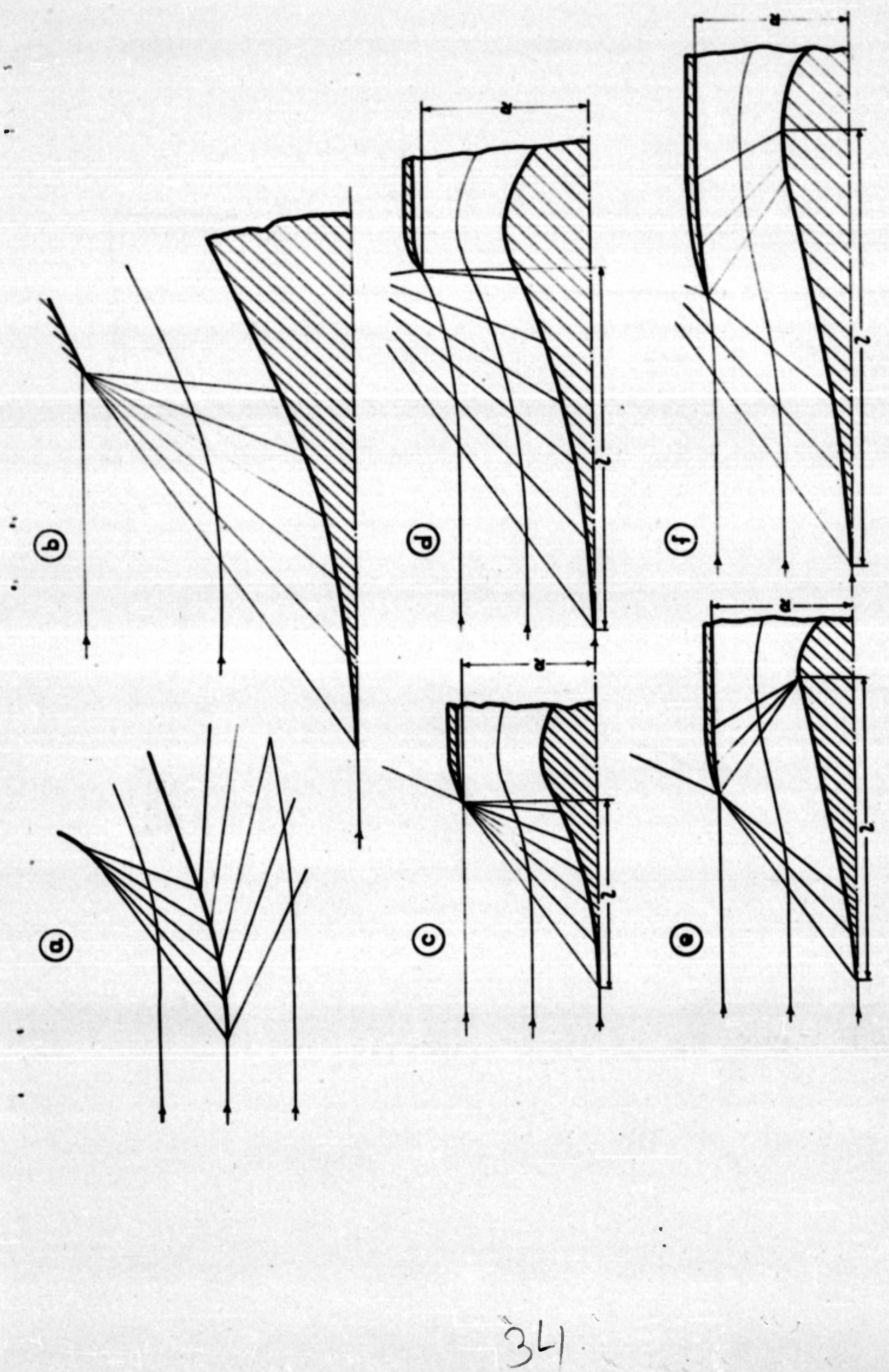
FIG. 10.



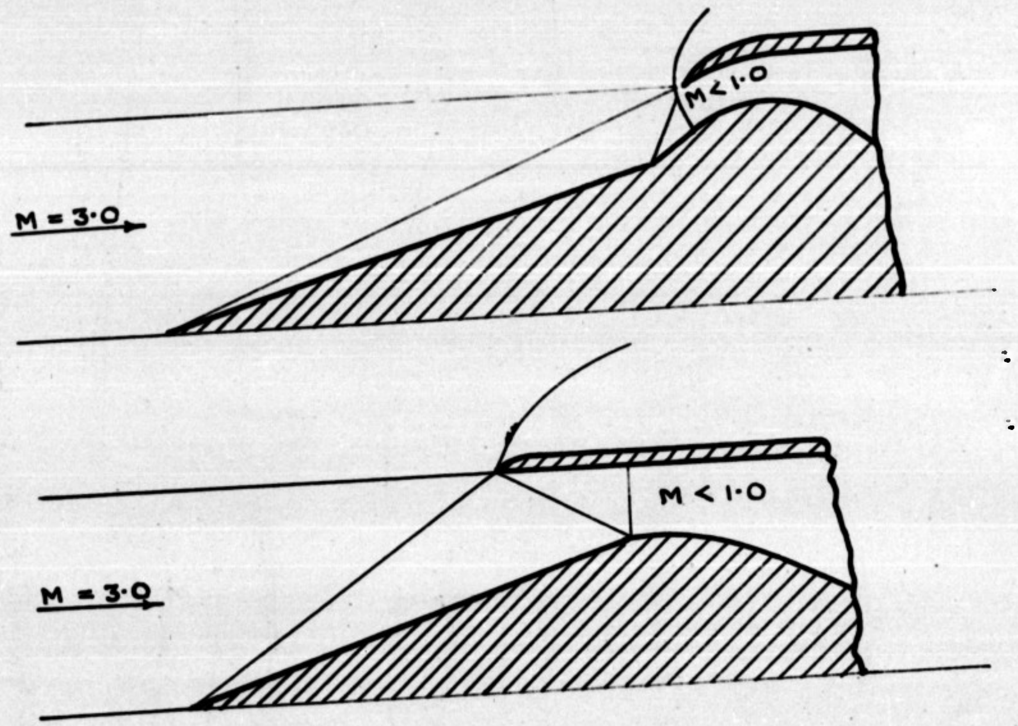
PRESSURE RECOVERY OF SINGLE-SHOCK SUPERSONIC DIFFUSERS (γ = 1·4)

39

FIG. 11.



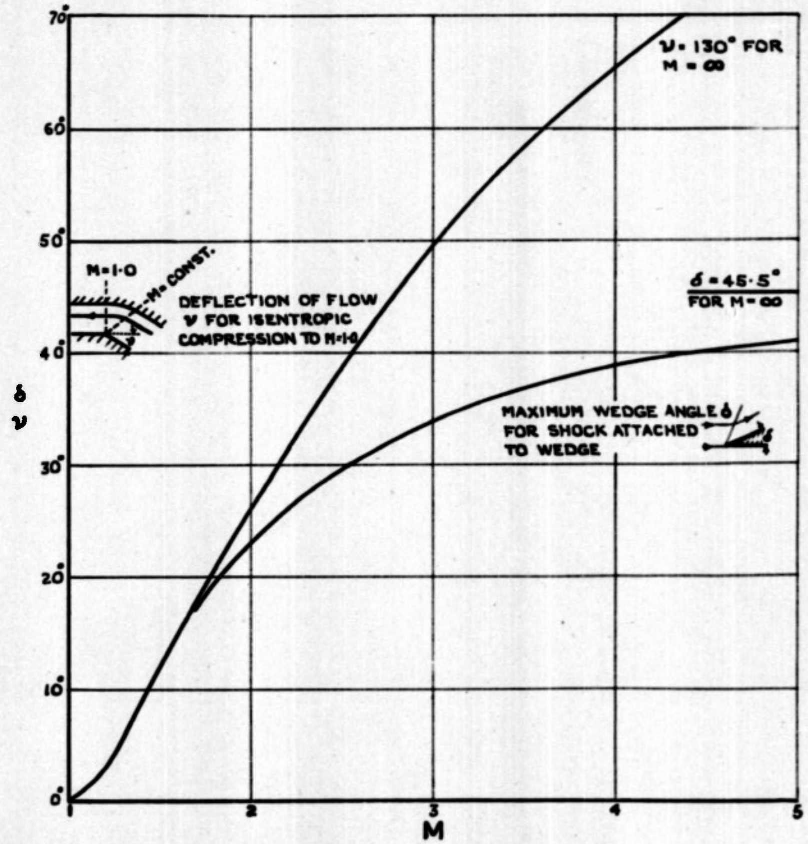
GEOMETRY OF MULTI-SHOCK DIFFUSER DESIGN



MULTI-SHOCK DIFFUSERS

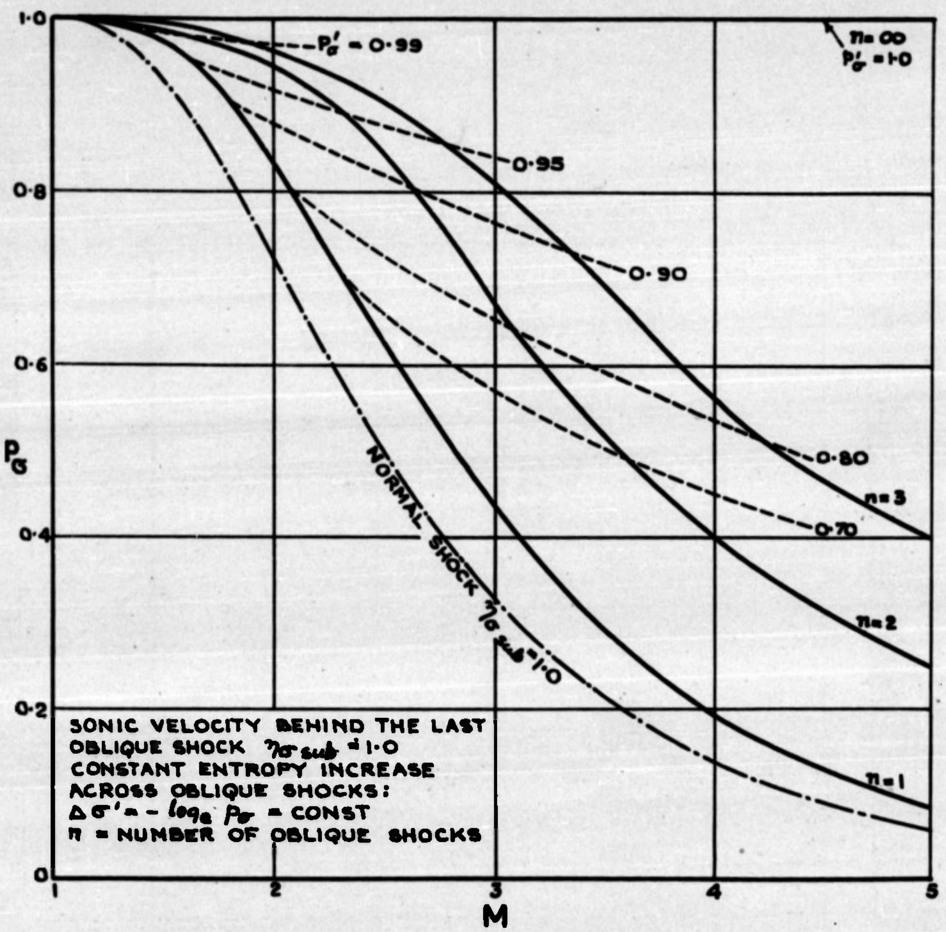
36.

FIG. 13.



DEFLECTION OF SUPERSONIC FLOW ν AND CRITICAL WEDGE ANGLE δ
($\gamma = 1{\cdot}4$)

FIG. 14.

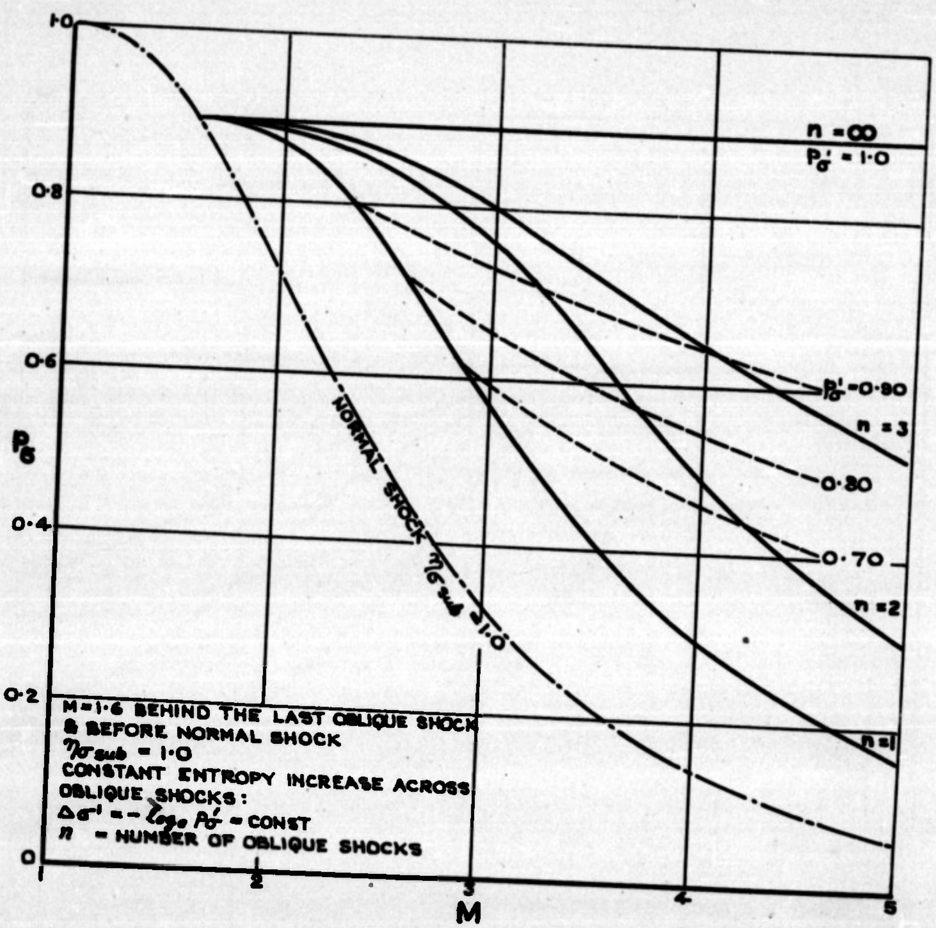


PRESSURE RECOVERY OF MULTI-SHOCK TWO-DIMENSIONAL SUPERSONIC DIFFUSERS
($\gamma = 1{\cdot}4$)

GAS. 5573.

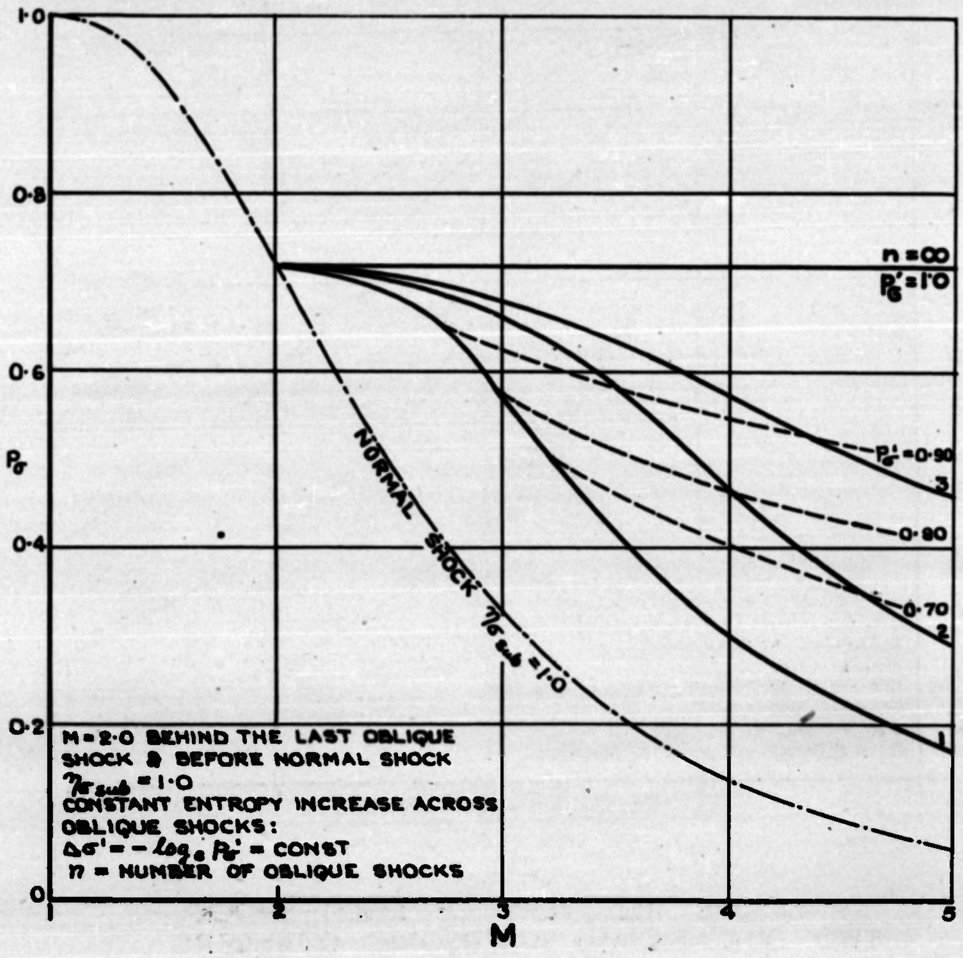
REPORT GAS. 8.

FIG. 15.



PRESSURE RECOVERY OF MULTI-SHOCK TWO-DIMENSIONAL SUPERSONIC DIFFUSERS (γ = 1·4)

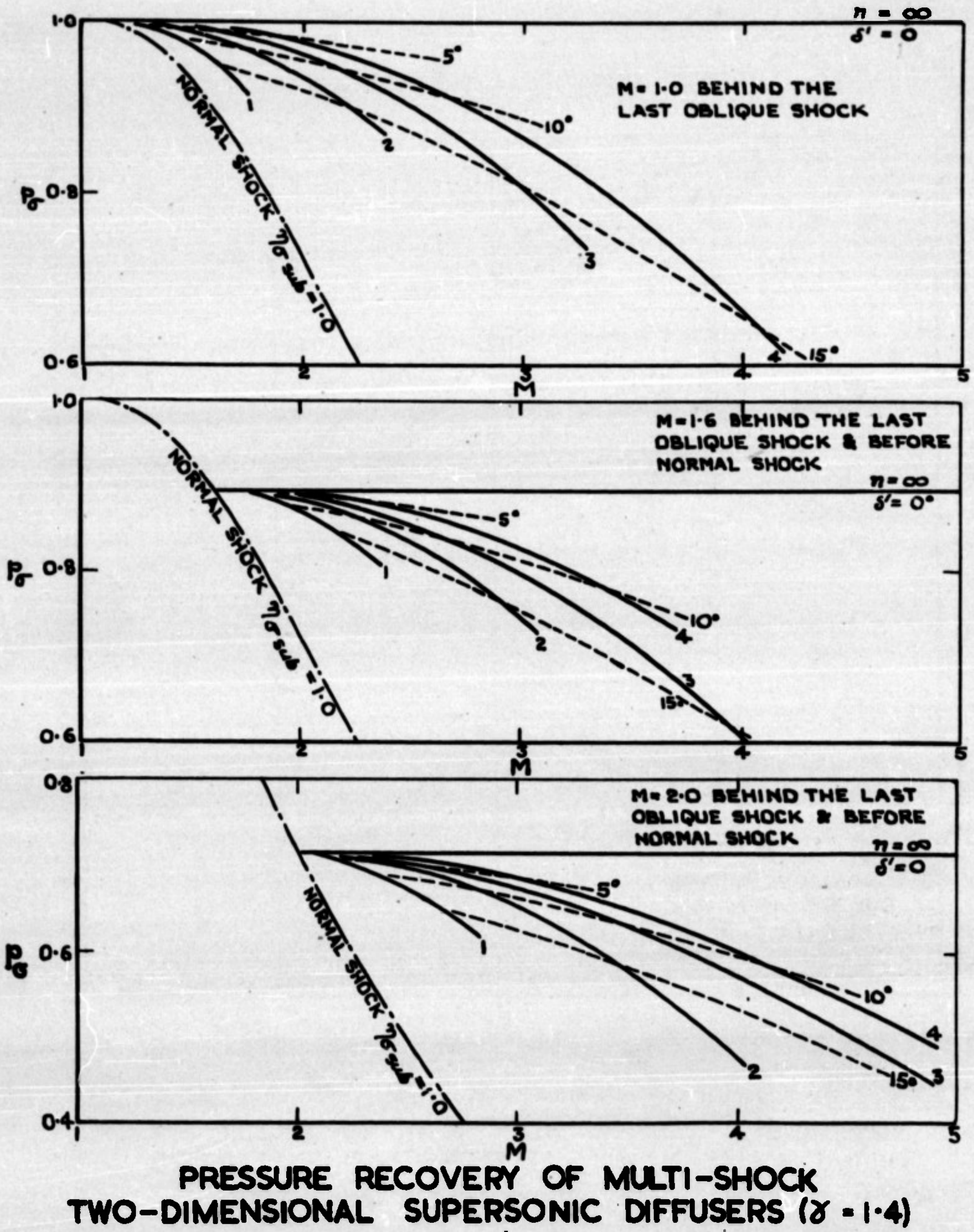
FIG. 16.



PRESSURE RECOVERY OF MULTI-SHOCK TWO-DIMENSIONAL SUPERSONIC DIFFUSERS

($\gamma = 1·4$)

FIG. 17.

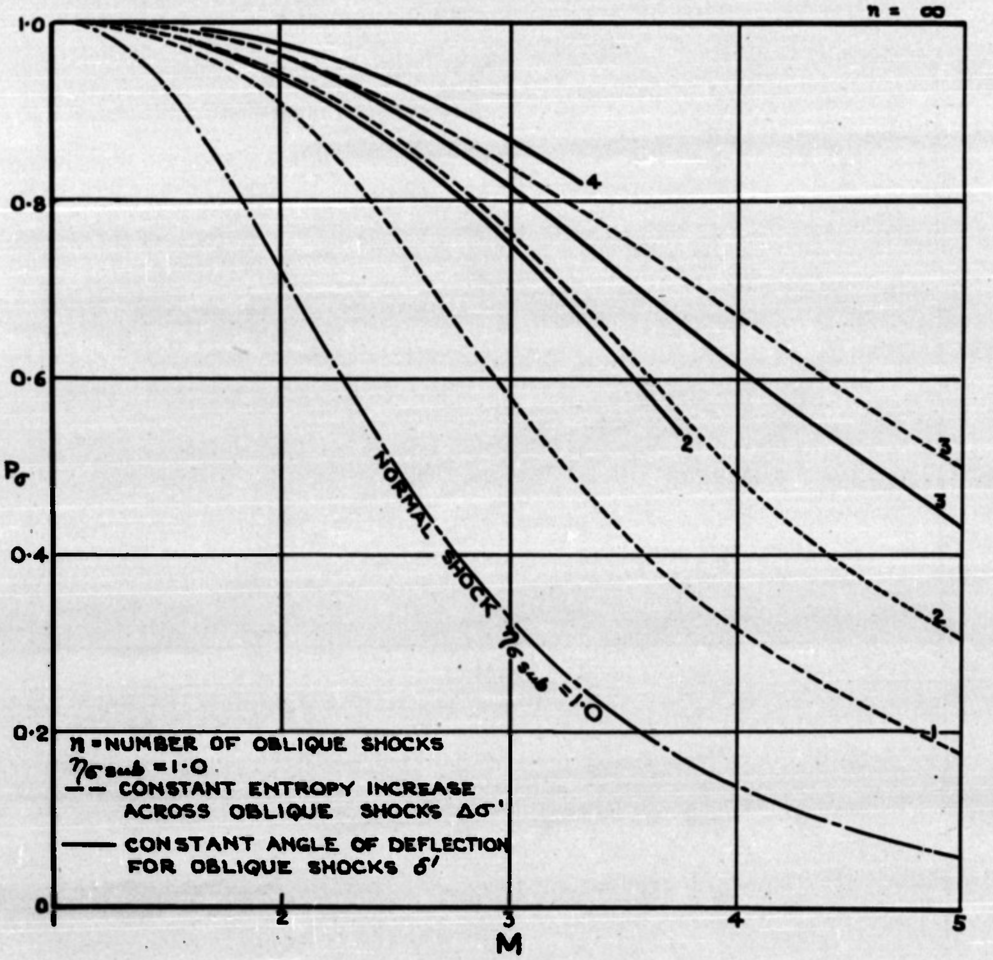


PRESSURE RECOVERY OF MULTI-SHOCK TWO-DIMENSIONAL SUPERSONIC DIFFUSERS ($\gamma = 1·4$)

CONSTANT ANGLE OF DEFLECTION δ' THROUGH OBLIQUE SHOCKS

n = No. OF OBLIQUE SHOCKS

FIG. 18.



MAXIMUM PRESSURE RECOVERY OF MULTI-SHOCK TWO-DIMENSIONAL SUPERSONIC DIFFUSERS ($\gamma = 1{\cdot}4$)

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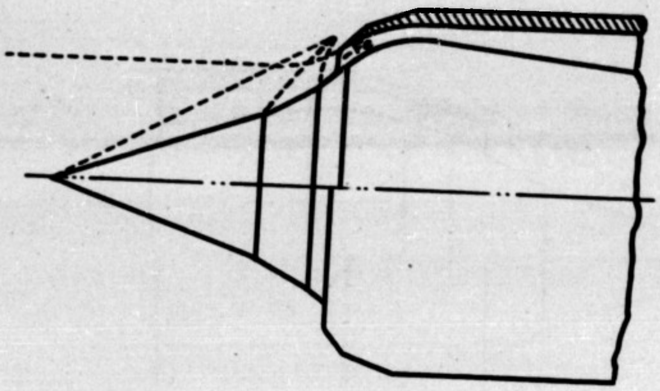
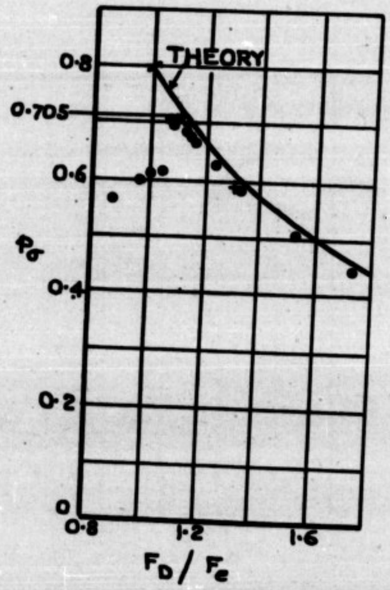
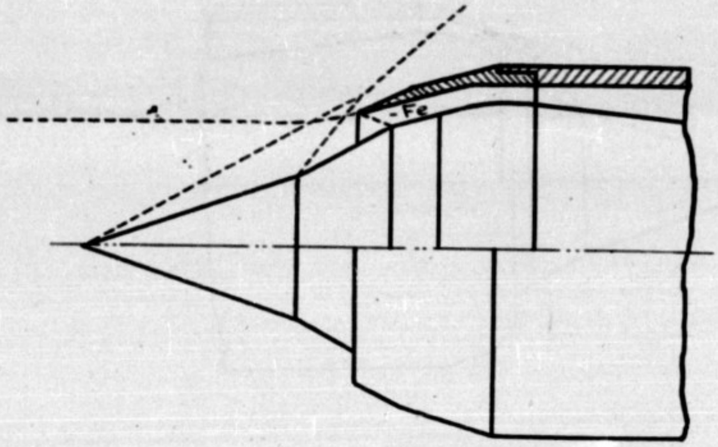


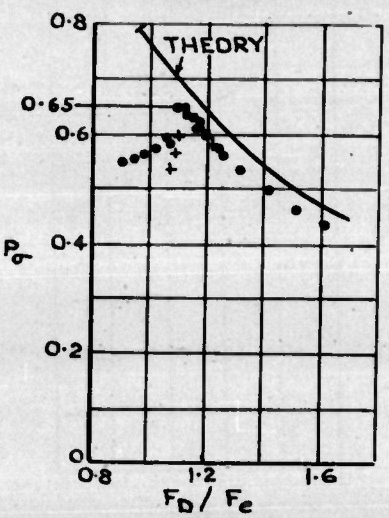


• - INCIDENCE $\alpha = 0°$ } M = 2·9
+ - " " $\alpha = 5°$ }
F_D - ADJUSTABLE DIFFUSER EXIT AREA
F_e - DIFFUSER ENTRY AREA

OSWATITSCH'S MULTI-SHOCK DIFFUSER No. III
(REF. 10)
DESIGN MACH NUMBER = 2·9

FIG.20





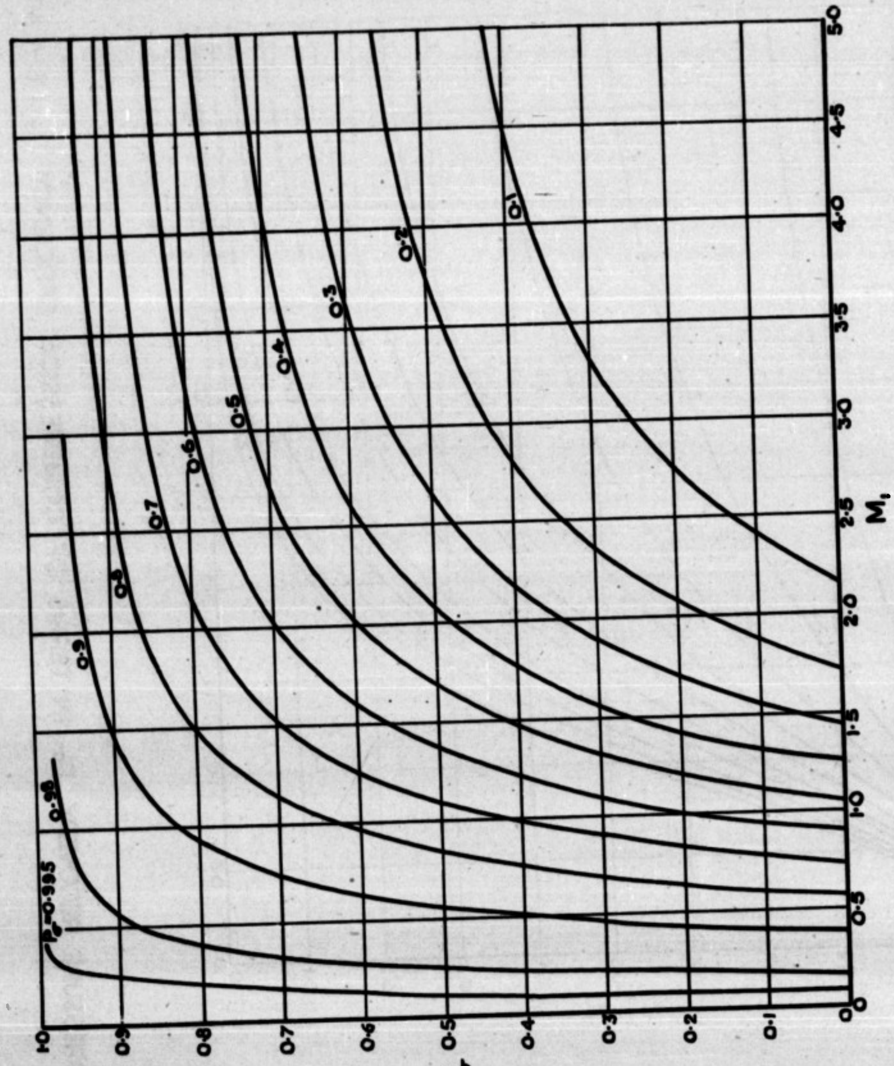
• - INCIDENCE $\alpha = 0°$ } M = 2·9
+ - " " $\alpha = 4·5°$

F_D - ADJUSTABLE DIFFUSER EXIT AREA
F_e - DIFFUSER ENTRY AREA

OSWATITSCH'S MULTI-SHOCK DIFFUSER No. VI
(REF. 10)
DESIGN MACH NUMBER = 2·9

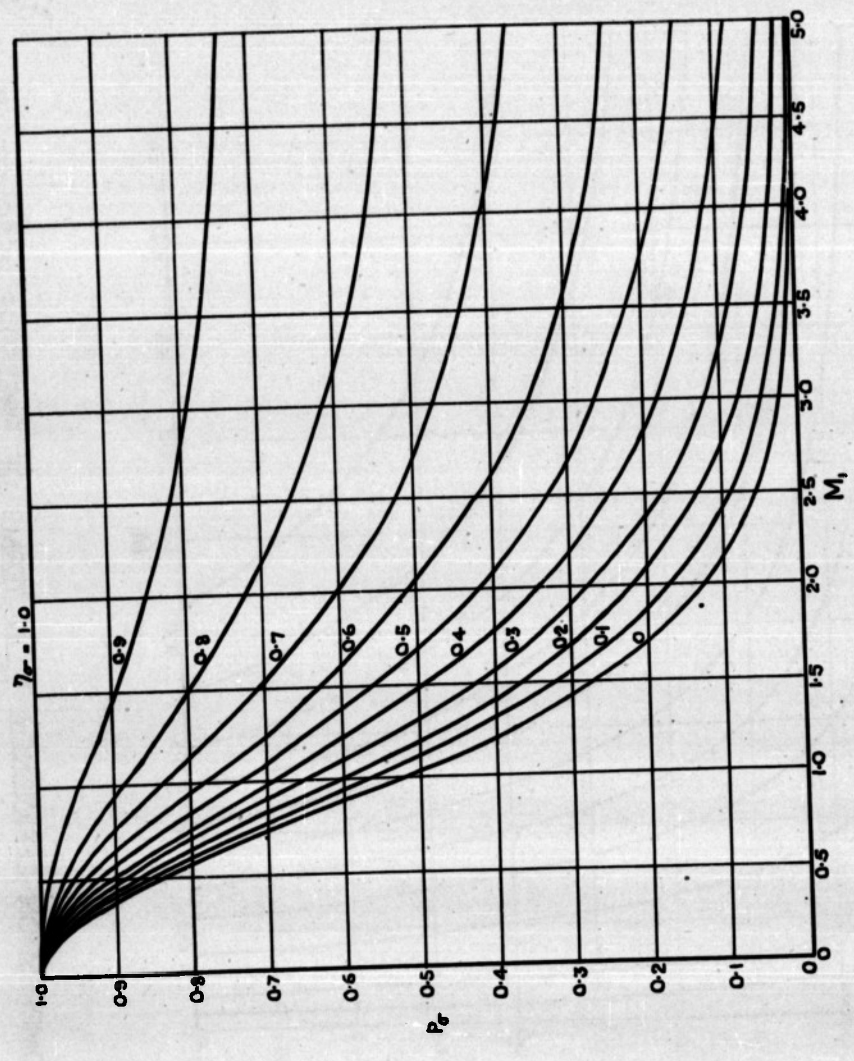
413

FIG. 21.



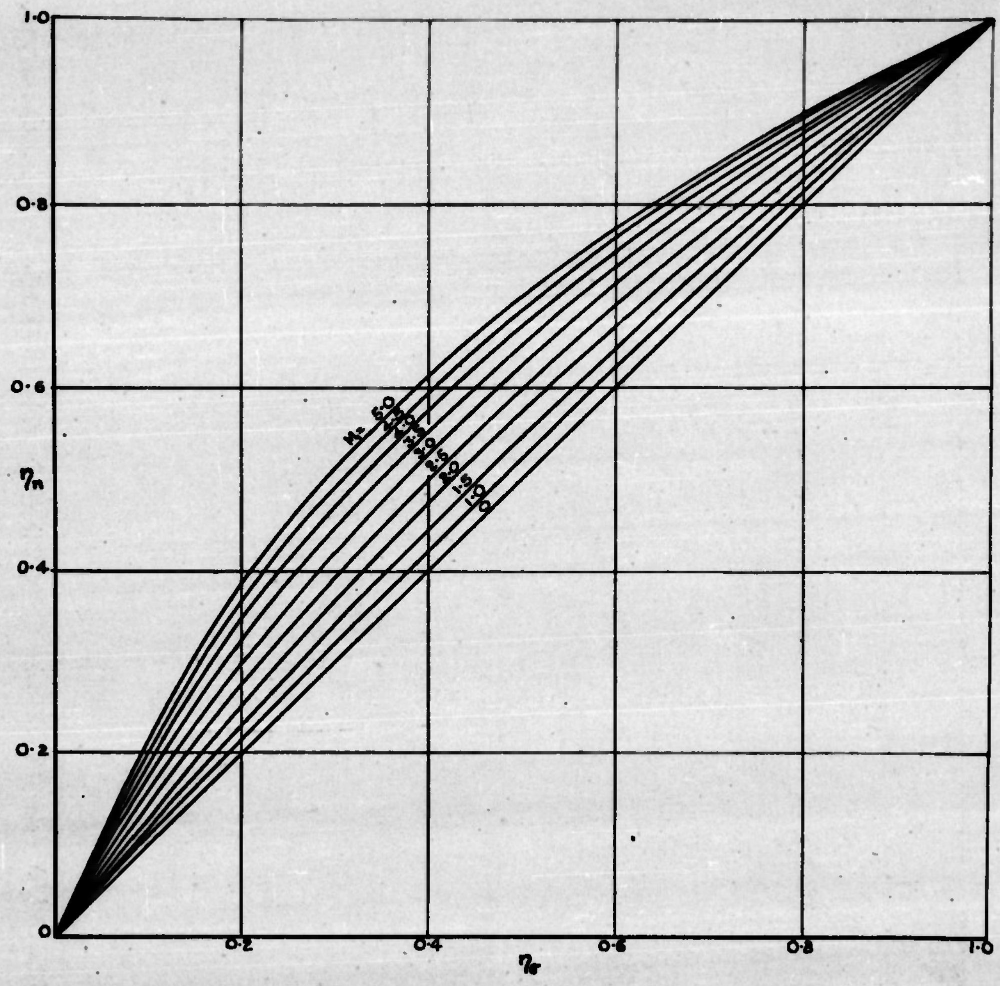
COMPRESSION EFFICIENCY η_σ IN TERMS OF PRESSURE RECOVERY p_σ (γ = 1·4000)

FIG. 22.



PRESSURE RECOVERY P_σ IN TERMS OF COMPRESSION EFFICIENCY η_σ (γ = 1·4000)

41



RELATION BETWEEN ISENTROPIC (η_σ) AND POLYTROPIC (η_n)
COMPRESSION EFFICIENCIES (γ = 1·4000)

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9726

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TITLE: Supersonic Diffusers

AUTHOR(S): Lukasiewicz, J.
ORIGINATING AGENCY: Royal Aircraft Establishment, Farnborough, Hants
PUBLISHED BY: (Same)

ATI- 9726
REVISION (None)
ORIG. AGENCY NO. RAE-Gas 8
PUBLISHING AGENCY NO. (Same)

DATE	DOC. CLASS.	COUNTRY	LANGUAGE	PAGES	ILLUSTRATIONS
June '46	Restr.	Gt. Brit.	Eng.	46	tables, graphs

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SUBJECT HEADINGS: Diffusors, Supersonic - Design (30204.4); Diffusors, Supersonic - Shock waves (30205.5)

ATI SHEET NO.: R-2-4-18

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